

# Turbulent Flows

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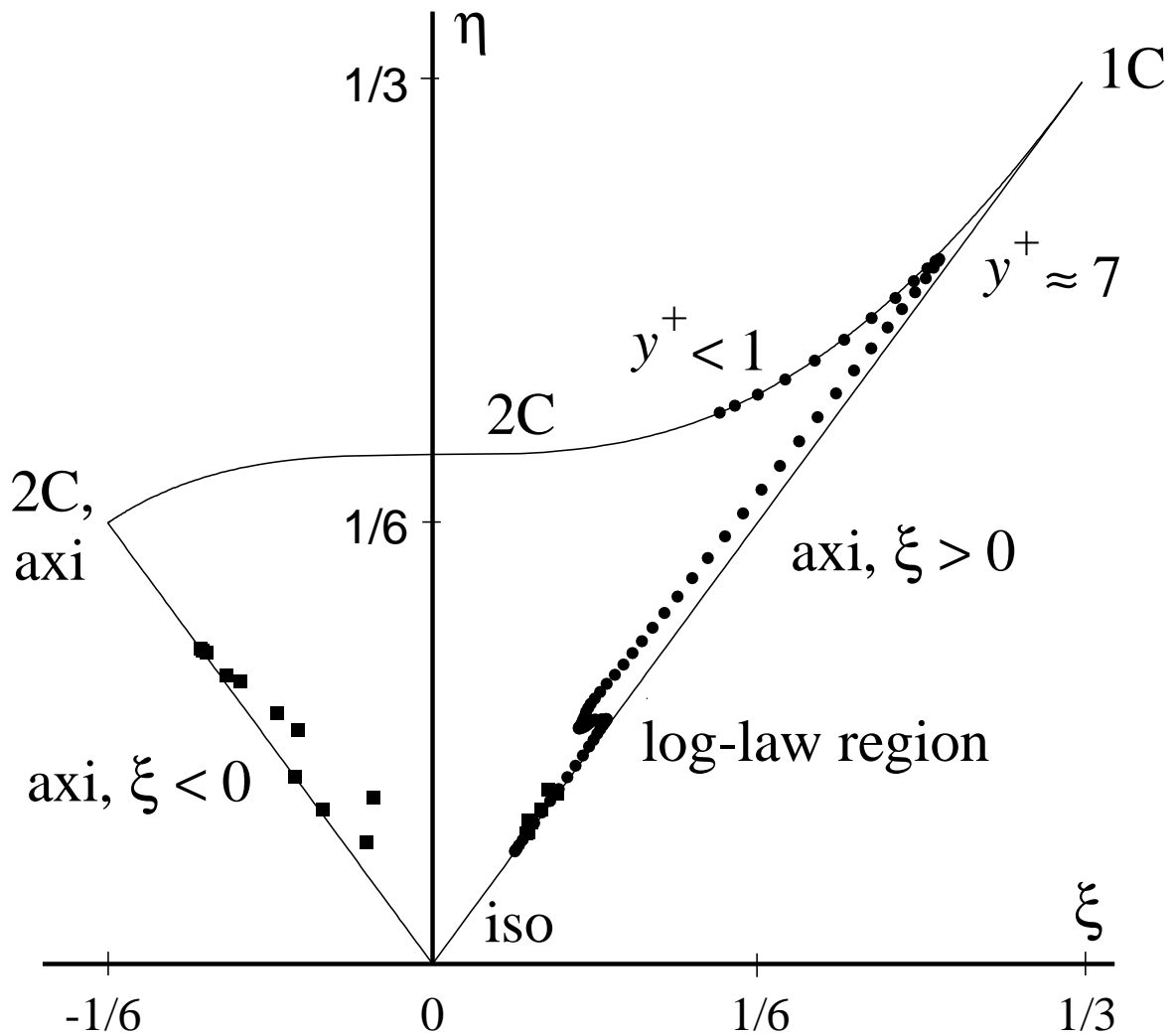


Figure 11.1: The Lumley triangle on the plane of the invariants  $\xi$  and  $\eta$  of the Reynolds-stress anisotropy tensor. The lines and vertices correspond to special states (see Table 11.1). Circles: from DNS of channel flow (Kim *et al.* 1987). Squares: from experiments on a turbulent mixing layer (Bell and Mehta 1990). 1C, one-component; 2C, two-component.

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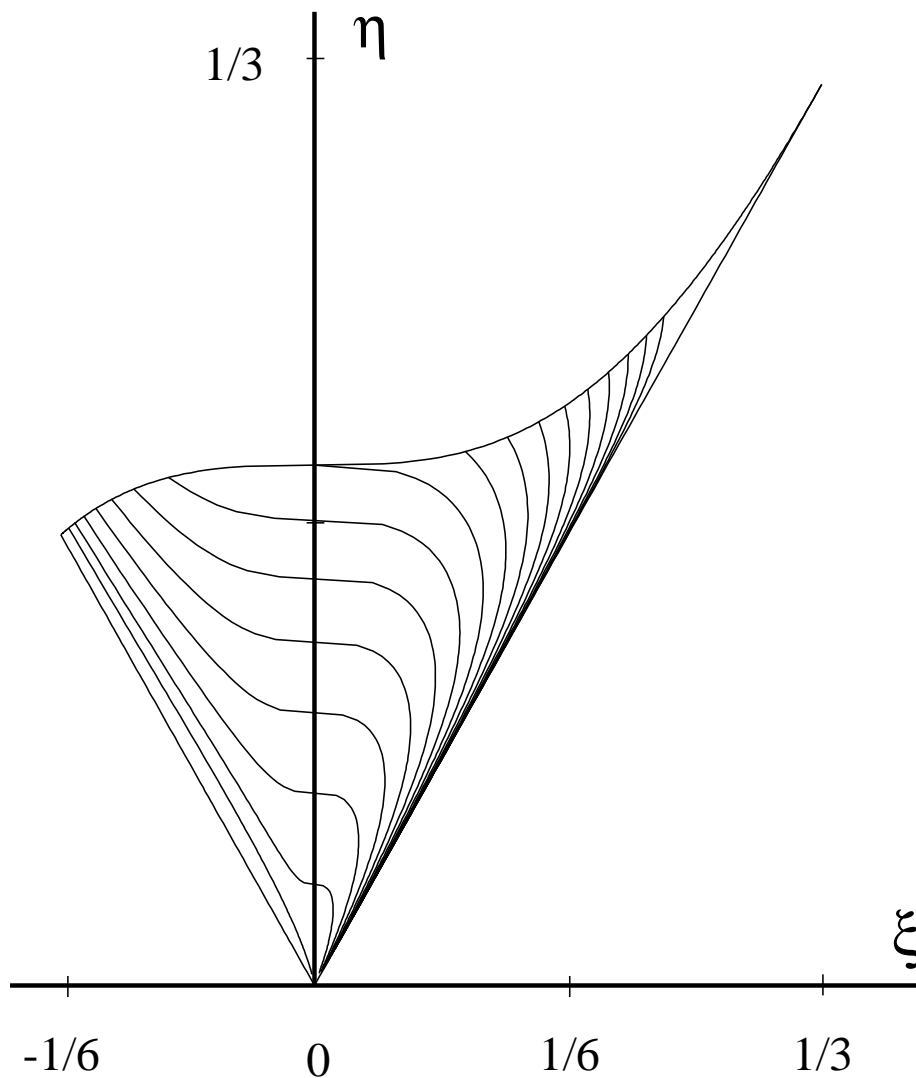


Figure 11.2: Trajectories on the  $\xi$ - $\eta$  plane given by the model of Sarkar and Speziale (1990) (Eqs. 11.51 and 11.57).

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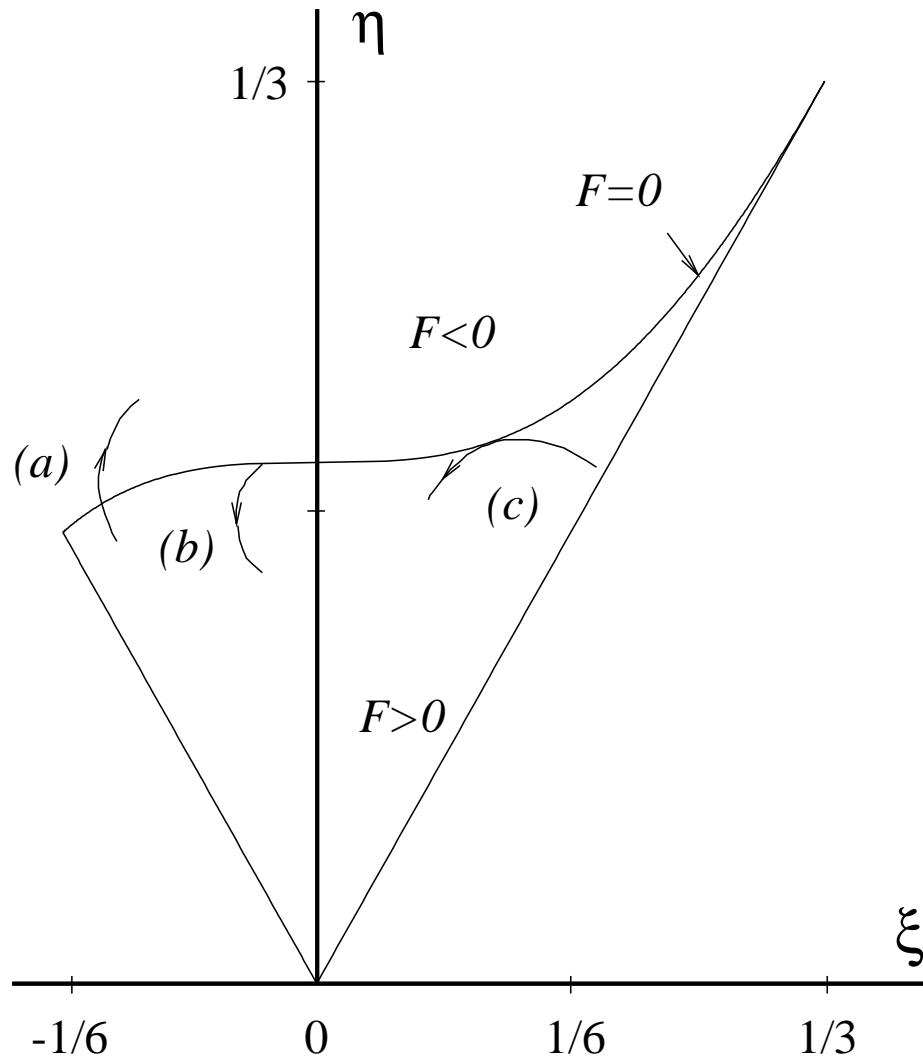


Figure 11.3: The Lumley triangle showing trajectories of three types: (a) violates realizability; (b) satisfies weak realizability; (c) satisfies strong realizability. (Note: other types of trajectories are possible.)

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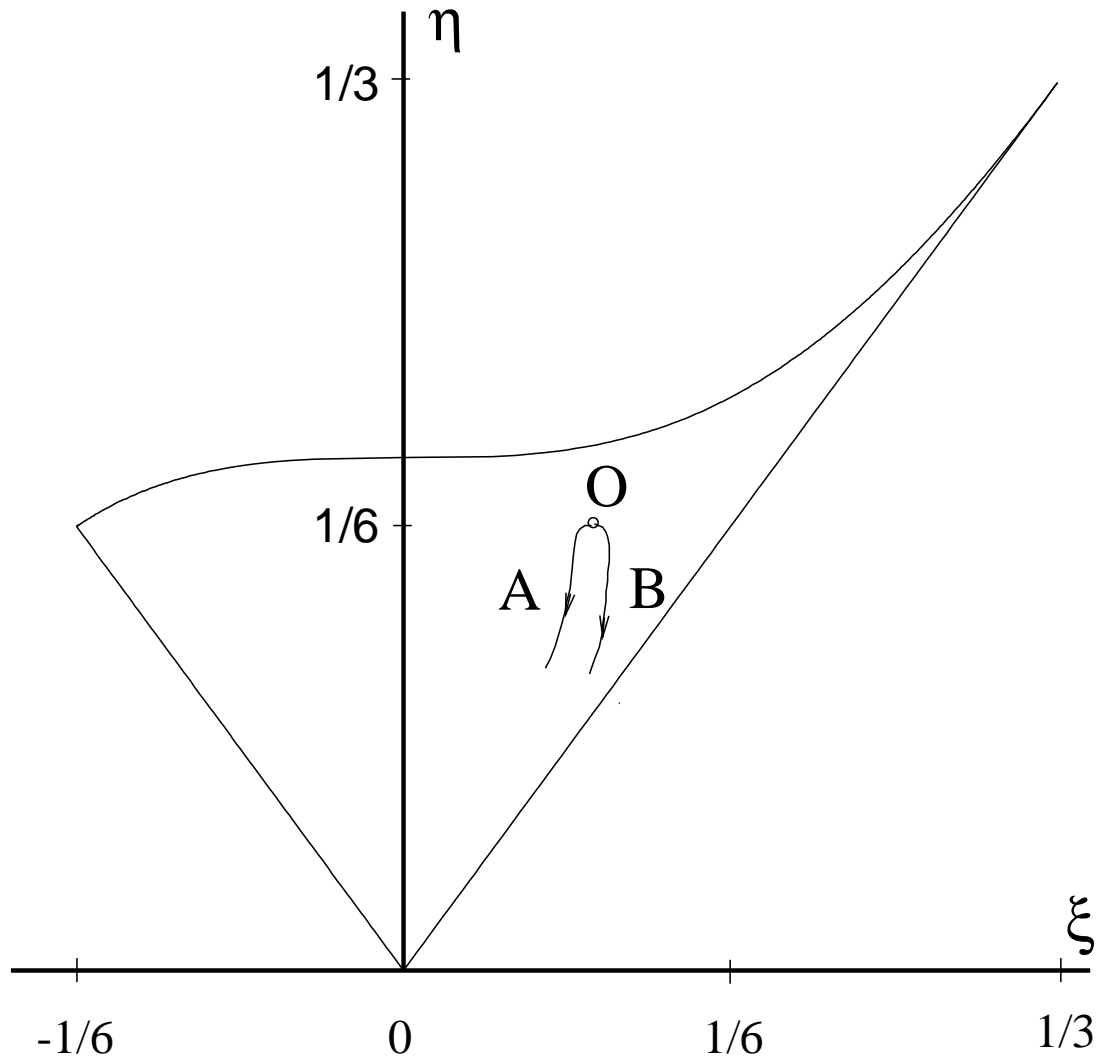


Figure 11.4: Sketch of trajectories (A and B) on the  $\xi$ - $\eta$  plane for two experiments (or DNS) in which the initial spectra are different, but the initial values of  $\mathbf{b}$  are the same. A Reynolds-stress model yields a unique trajectory from initial point  $O$ .

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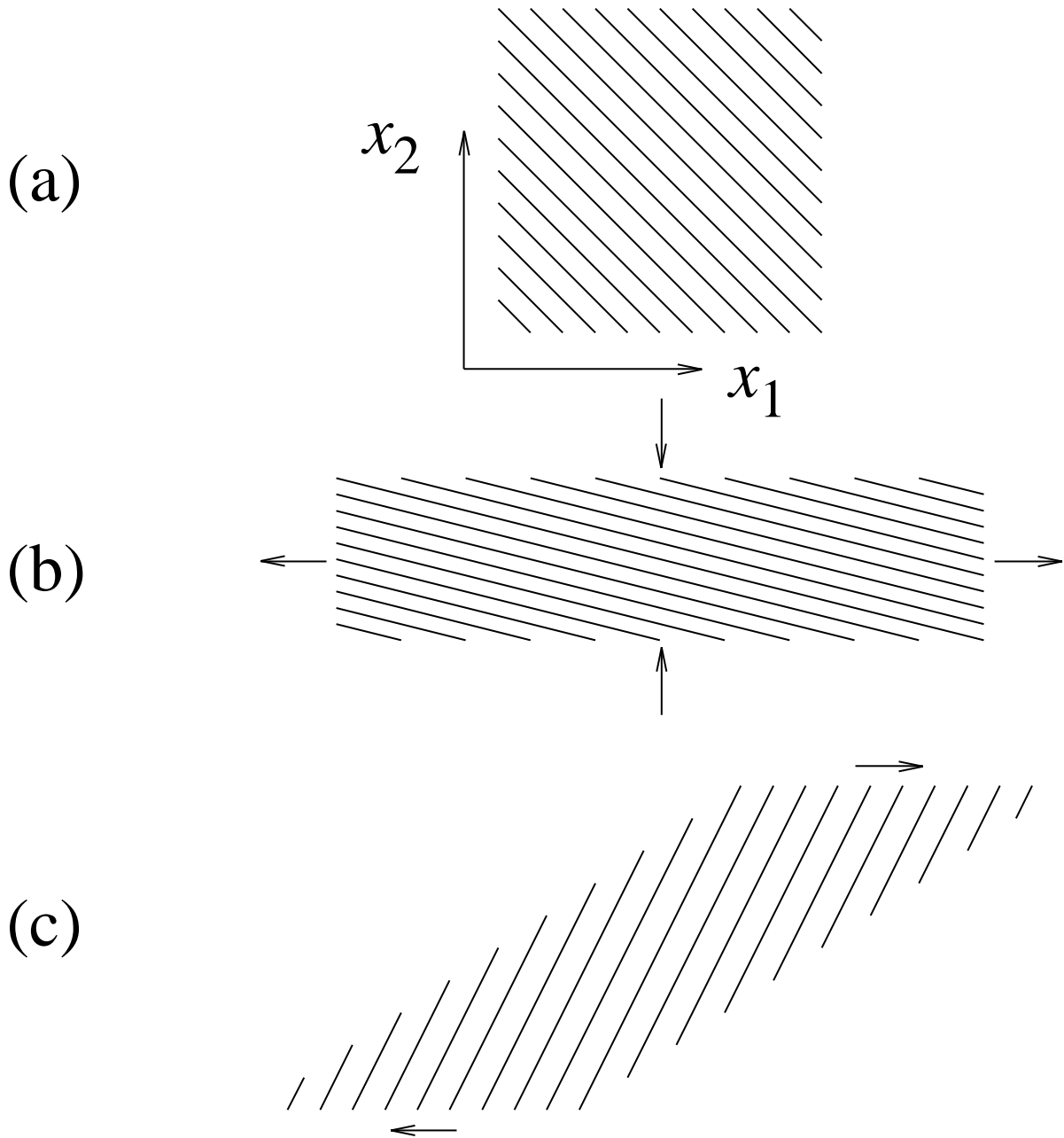


Figure 11.5: Crests of the fields  $\phi(\mathbf{x}, t)$  evolving by  $\bar{D}\phi/\bar{D}t = 0$  (a) initial condition,  $\phi e^{i\mathbf{k}^o \cdot \mathbf{x}}$ ,  $\kappa_1^o = \kappa_2^o > 0$ ,  $\kappa_3^o = 0$  (b) after plane straining ( $\bar{S}_{11} = -\bar{S}_{22} > 0$ ) (c) after shearing  $\partial\langle U_1 \rangle/\partial x_2 > 0$ .

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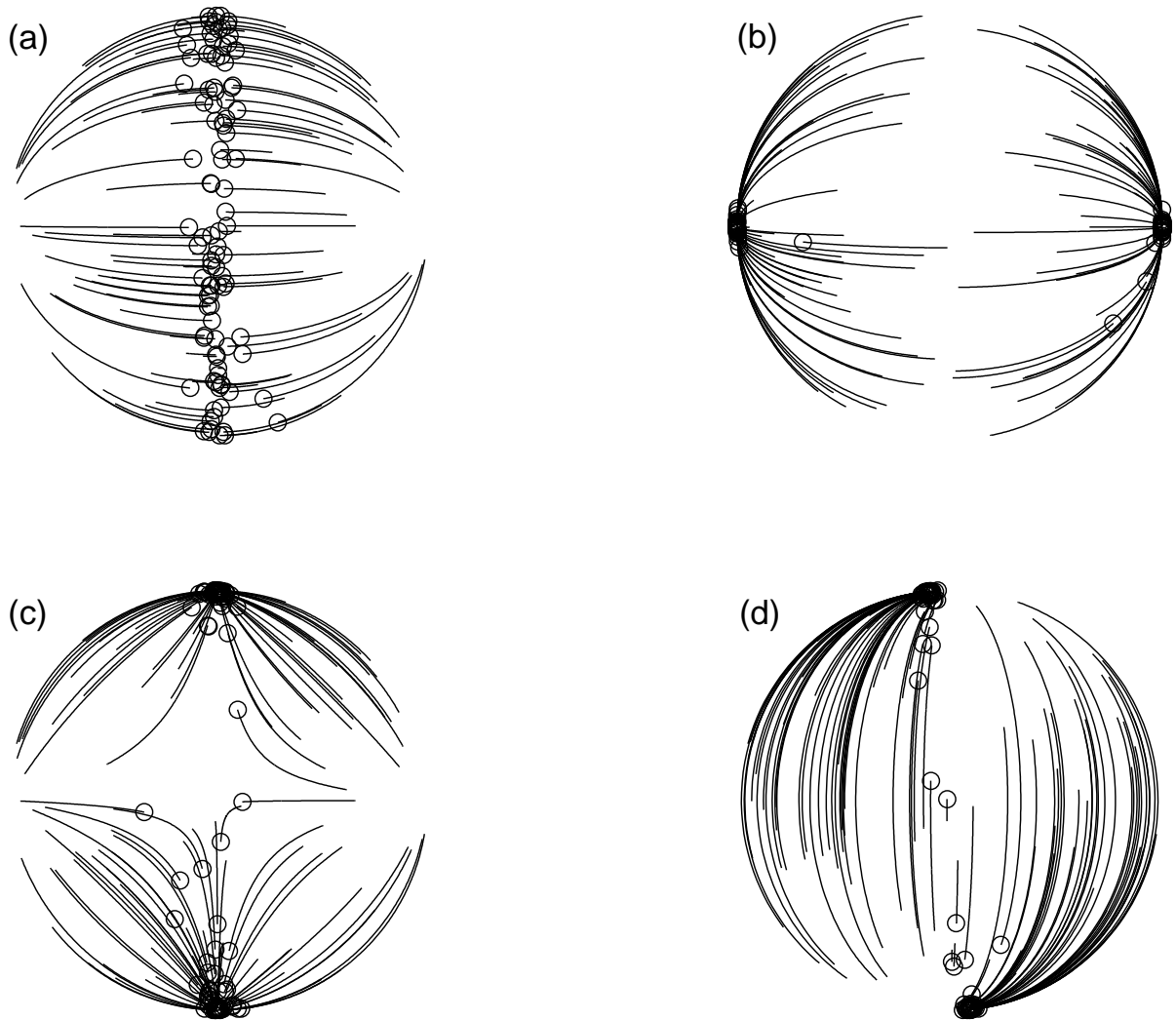


Figure 11.6: Trajectories of the unit wavevector  $\hat{e}(t)$  on the unit sphere from random initial conditions for (a) axisymmetric contraction (b) axisymmetric expansion (c) plane strain (d) shear. The  $\hat{e}_1$  direction is horizontal, the  $\hat{e}_2$  direction is vertical, and the  $\hat{e}_3$  direction is into the page. The symbols mark the ends of the trajectories after distortion.

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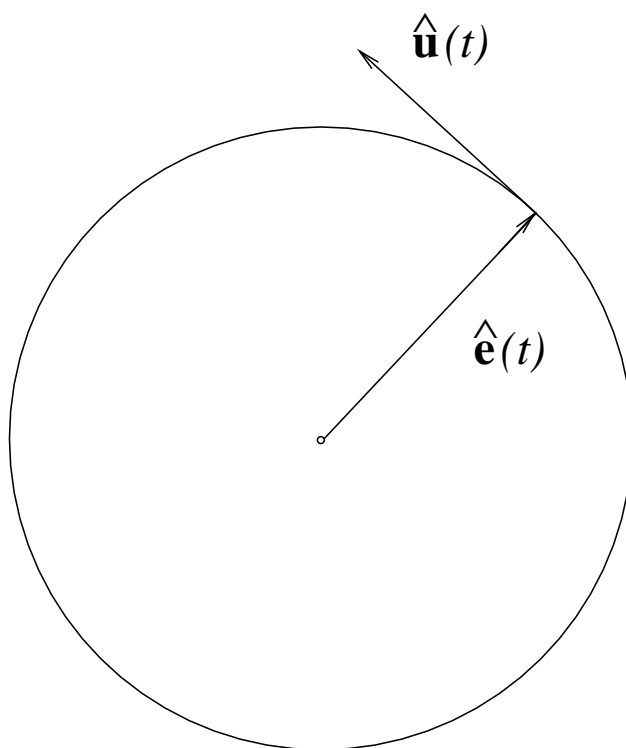


Figure 11.7: Sketch of the unit sphere showing the unit wavevector  $\hat{\mathbf{e}}(t)$ . The Fourier component of velocity  $\hat{\mathbf{u}}(t)$  is orthogonal to  $\hat{\mathbf{e}}(t)$ , and so it is in the tangent plane of the unit sphere at  $\hat{\mathbf{e}}(t)$ .

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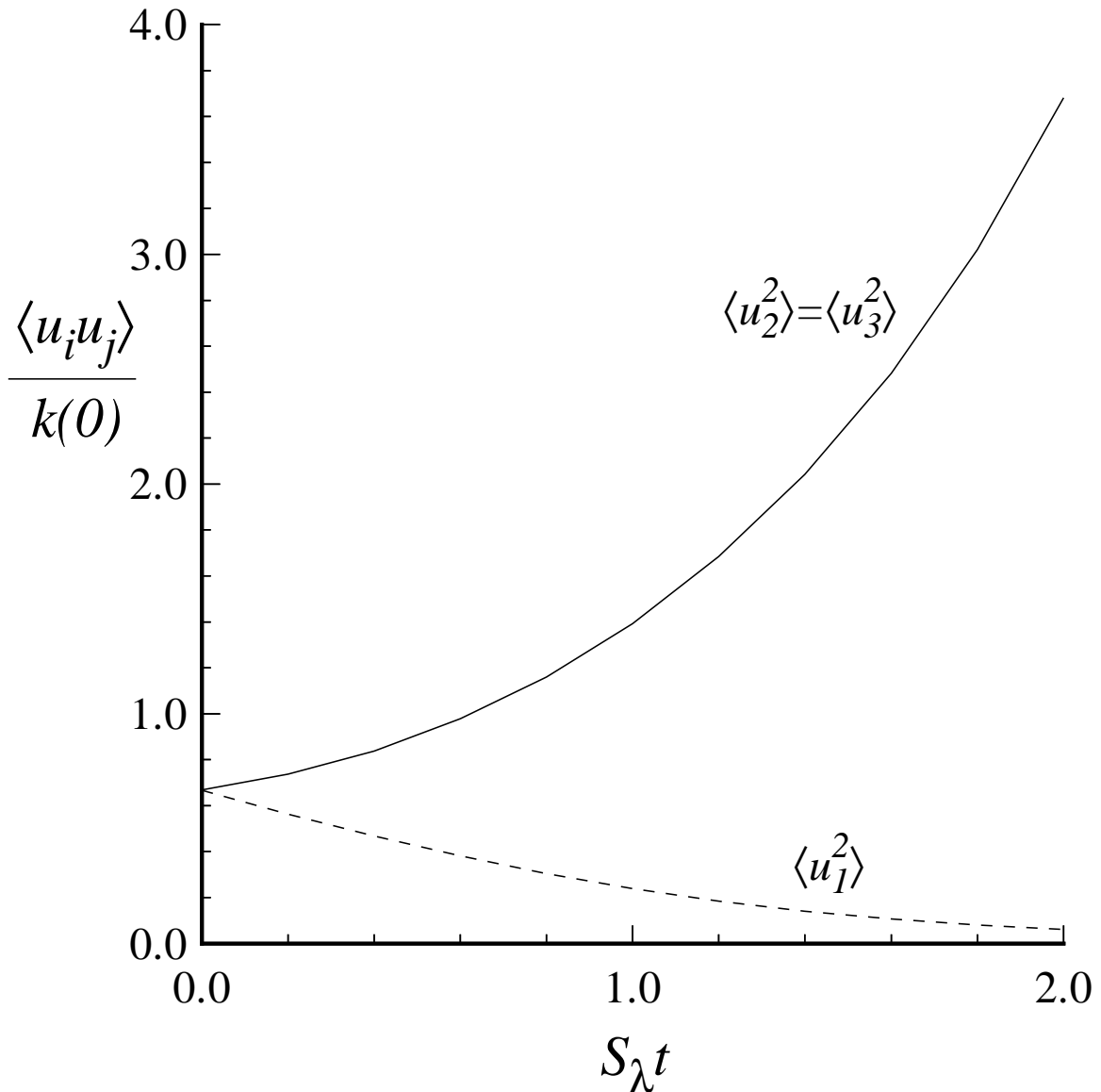


Figure 11.8: Sketch of the unit sphere showing the unit wavevector  $\hat{\mathbf{e}}(t)$ . The Fourier component of velocity  $\hat{\mathbf{u}}(t)$  is orthogonal to  $\hat{\mathbf{e}}(t)$ , and so it is in the tangent plane of the unit sphere at  $\hat{\mathbf{e}}(t)$ .



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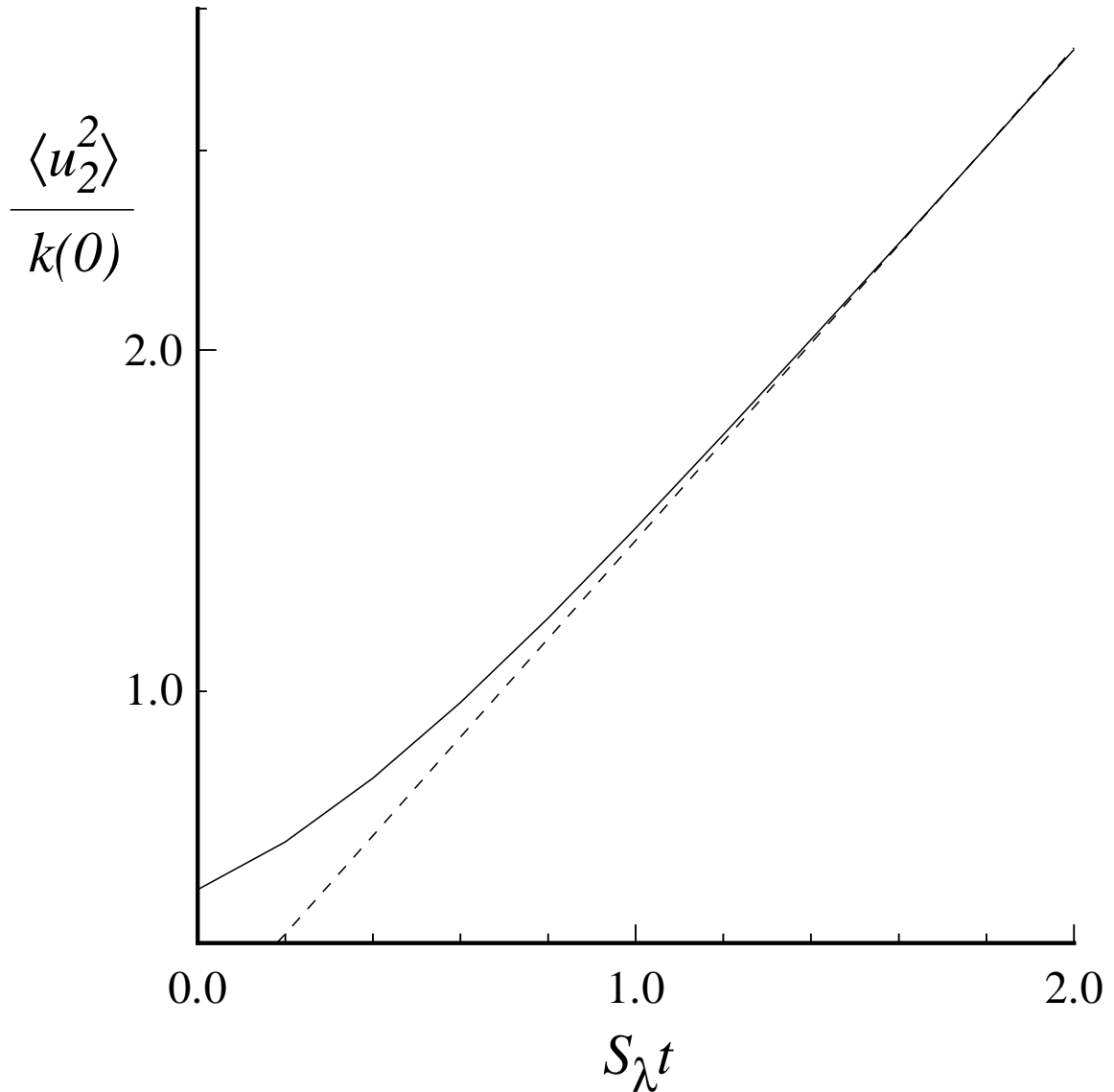


Figure 11.9: Evolution of  $\langle u_2^2 \rangle$  (on a log scale) for axisymmetric contraction rapid distortion (solid line). The dashed line is  $\frac{1}{2} \exp(S_\lambda t)$  indicating the asymptotic growth rate.

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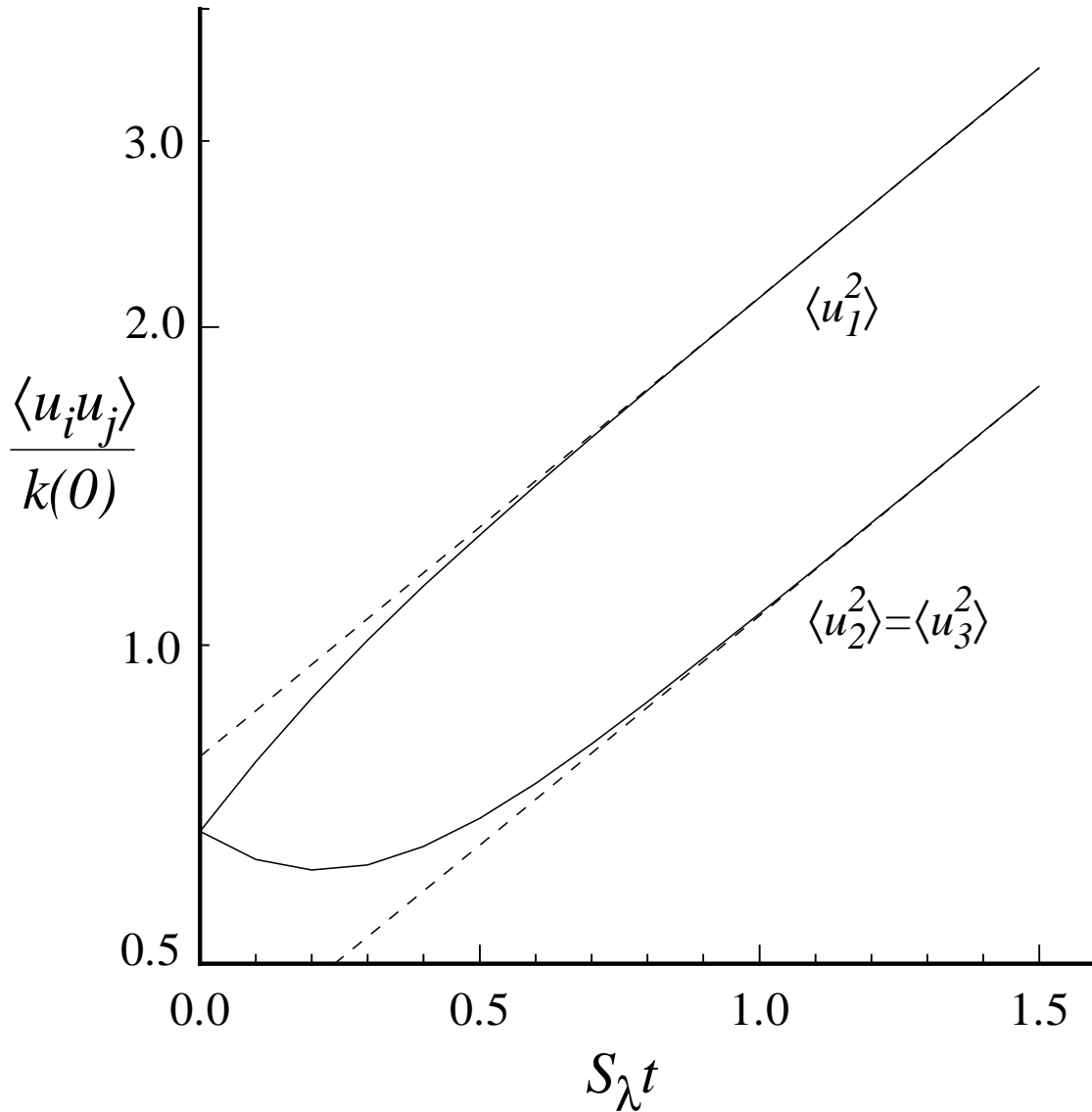


Figure 11.10: Evolution of Reynolds stresses for axisymmetric expansion rapid distortion. The dashed lines show the asymptotic growth as  $\exp(S_\lambda t)$ .

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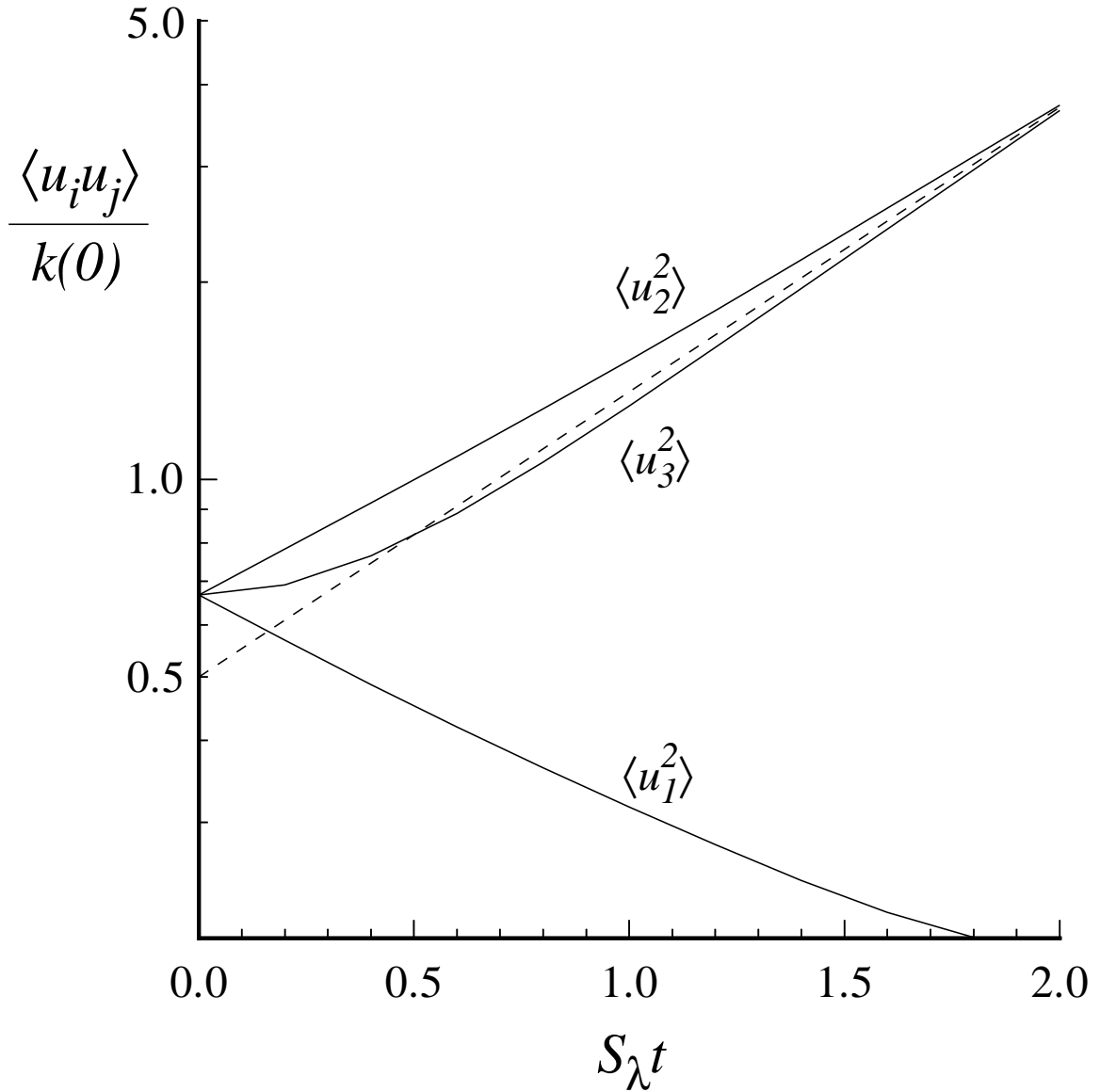


Figure 11.11: Evolution of Reynolds stresses for plane strain rapid distortion. The dashed line is  $\frac{1}{2} \exp(S_\lambda t)$ .

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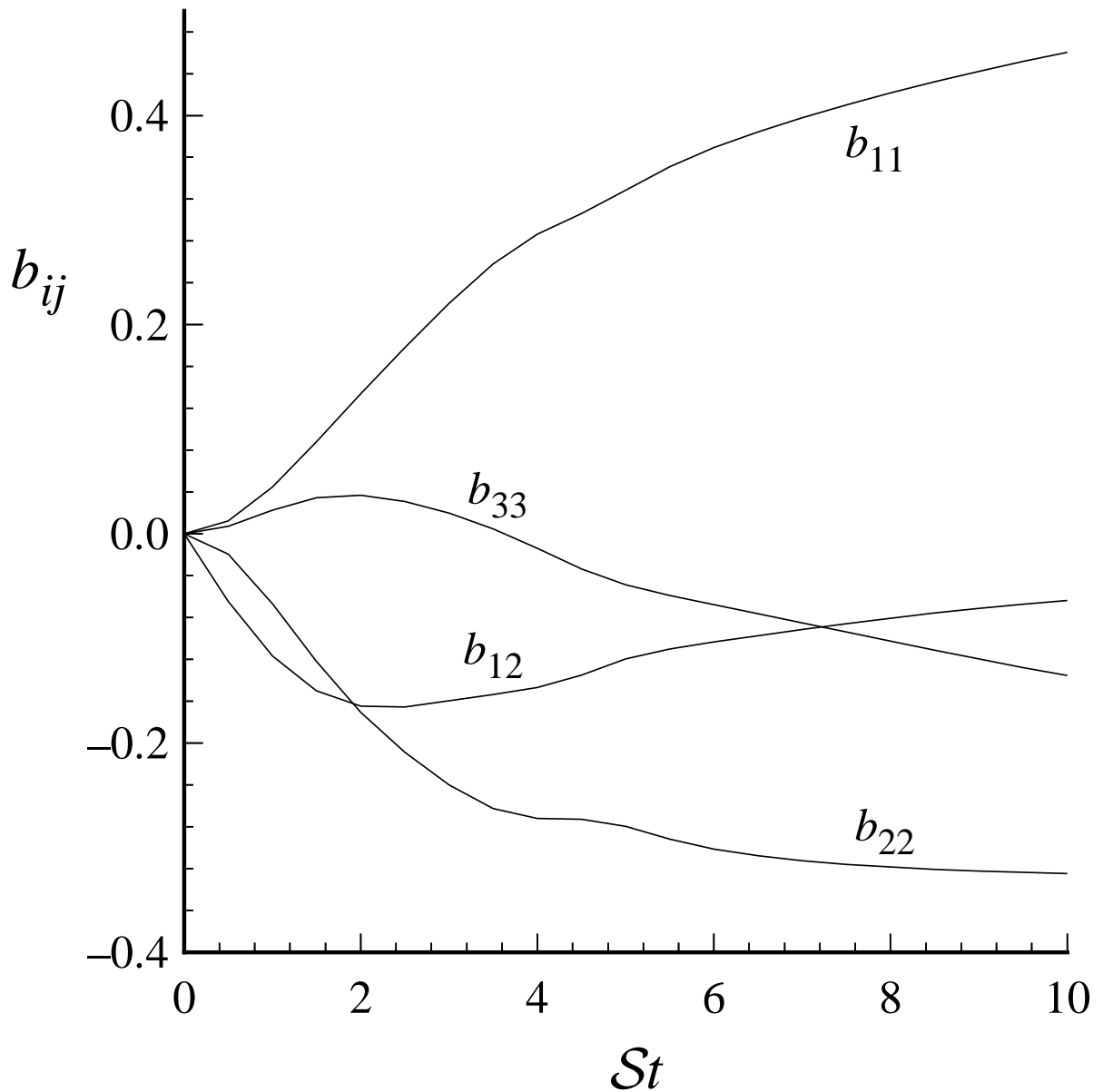


Figure 11.12: Evolution of Reynolds-stress anisotropies for shear rapid distortion.

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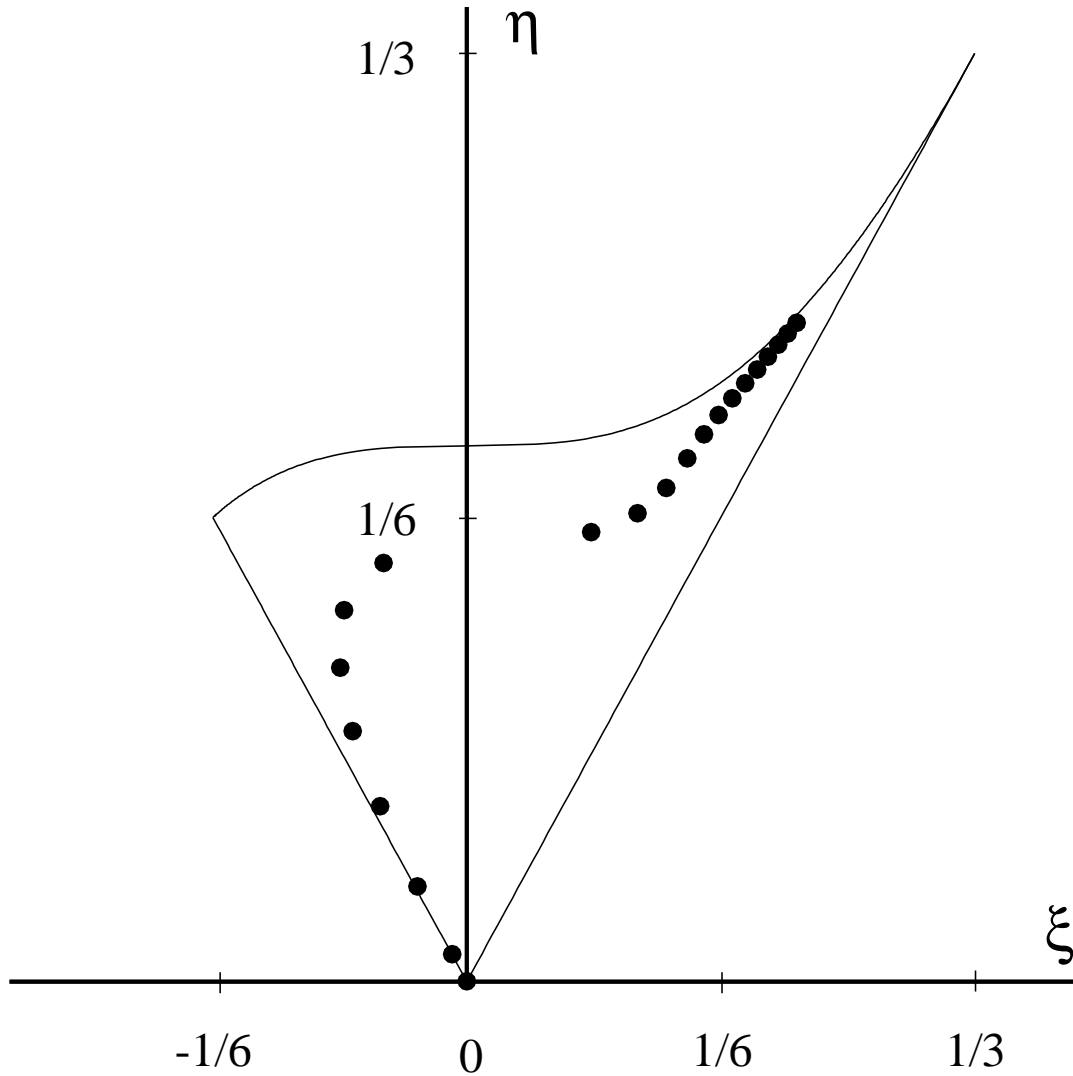


Figure 11.13: Evolution of the Reynolds-stress invariants for shear rapid distortion. Starting from the origin (corresponding to isotropy), each symbol gives the state after an amount of shear  $St = 0.5$ .

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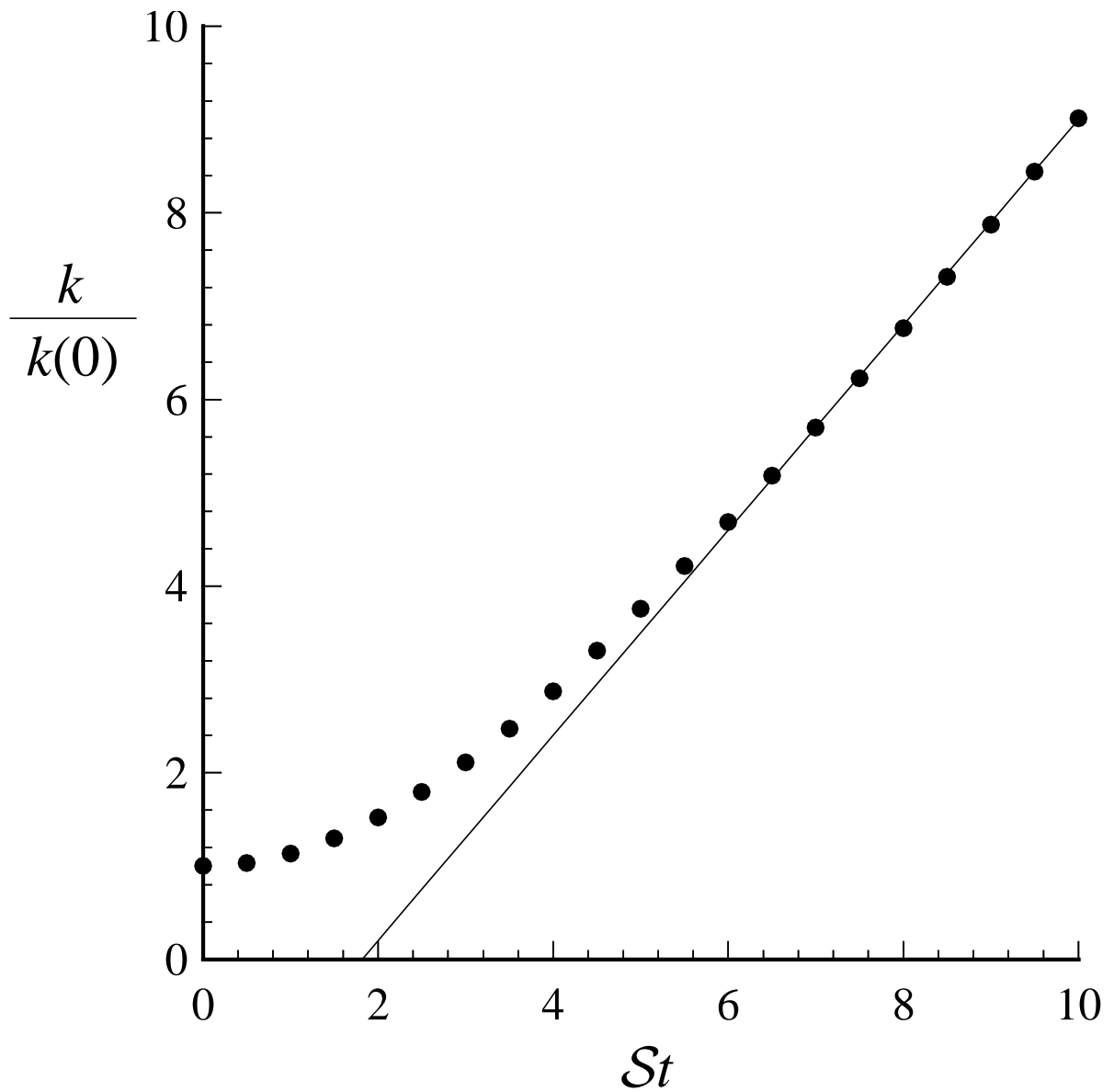


Figure 11.14: Evolution of the turbulent kinetic energy for shear rapid distortion.

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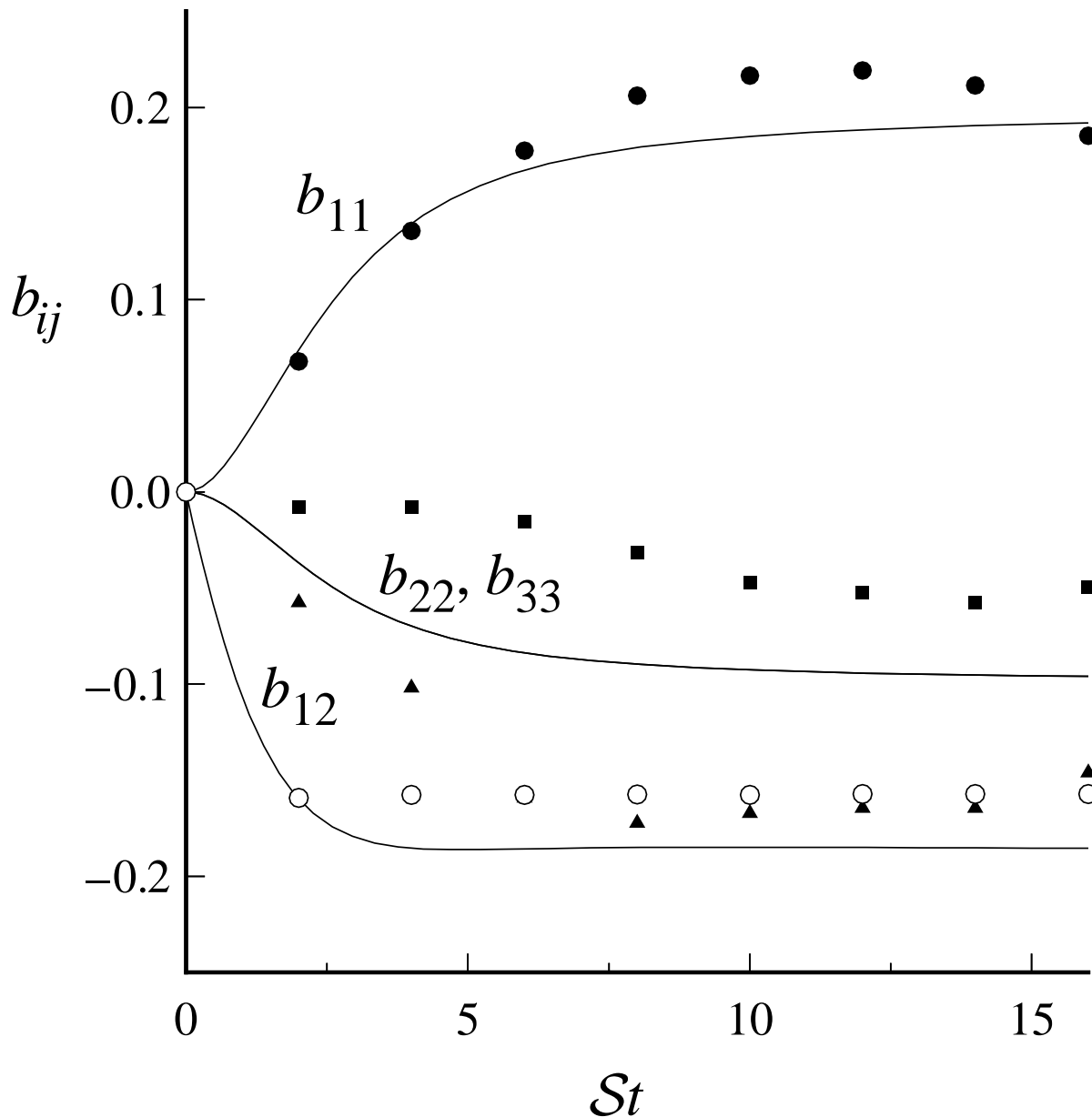


Figure 11.15: Reynolds-stress anisotropies in homogeneous shear flow. Comparison of LRR-IP model calculations (lines) with the DNS data of Rogers Moin (1987) (symbols):  $\bullet$ ,  $b_{11}$ ;  $\circ$ ,  $b_{12}$ ; squares,  $b_{22}$ ; triangles,  $b_{33}$ .

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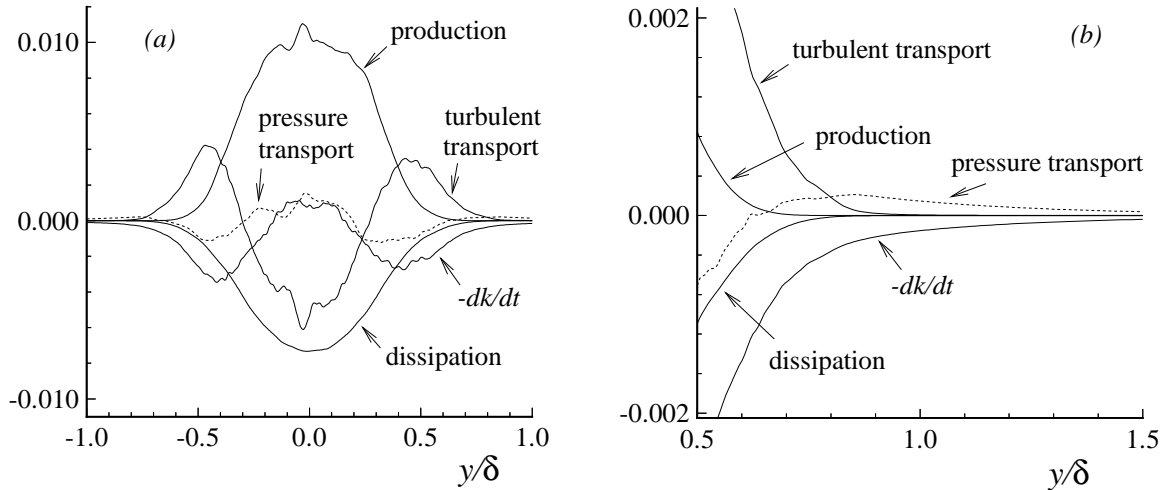


Figure 11.16: Kinetic energy budget in the temporal mixing layer from the DNS data of Rogers and Moin (1994): (a) across the whole flow (b) an expanded view of the edge of the layer. The contributions to the budget are: production  $\mathcal{P}$ ; dissipation  $-\varepsilon$ ; rate of change  $-dk/dt$ ; turbulent transport; pressure transport (dashed line). All quantities are normalized by the velocity difference and the layer thickness  $\delta$  (see Fig. 5.21).



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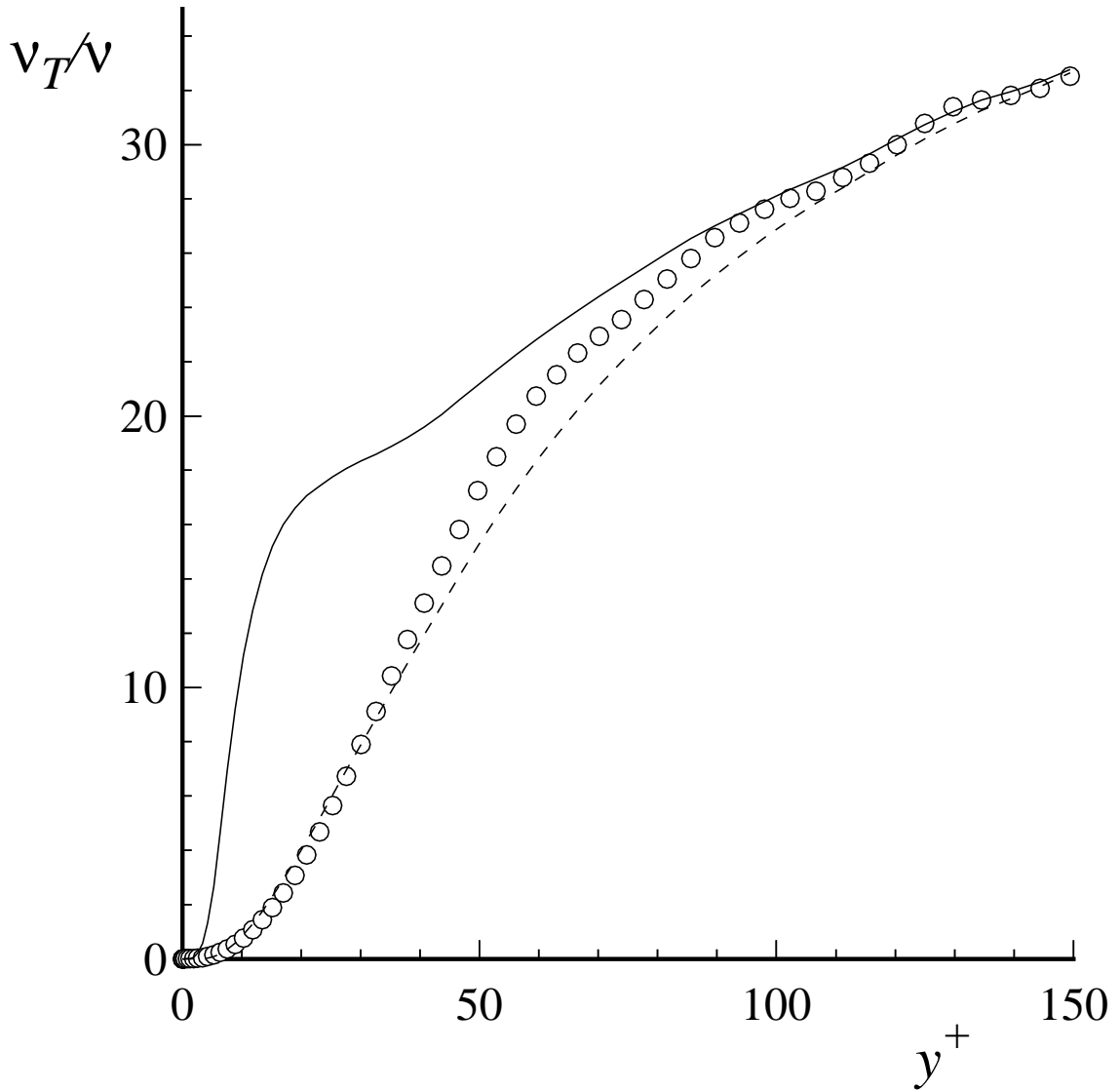


Figure 11.17: Turbulent viscosity against  $y^+$  for channel flow at  $Re = 13,750$ . Symbols, DNS data of Kim *et al.* (1987); solid line,  $0.09k^2/\varepsilon$ ; dashed line,  $0.22\langle v^2 \rangle k/\varepsilon$ .

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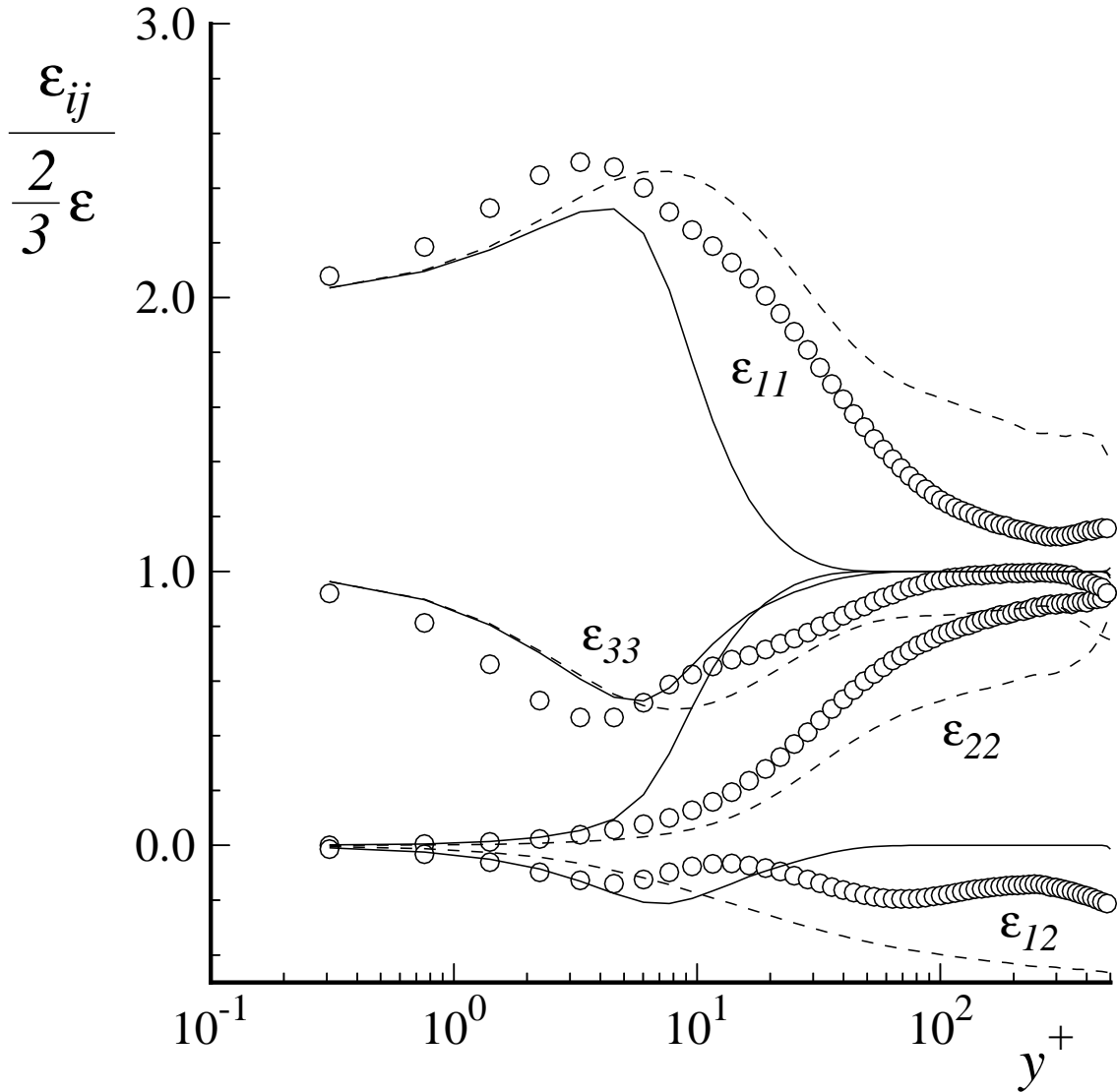


Figure 11.18: Normalized dissipation components in a turbulent boundary layer at  $Re_\theta = 1,410$ : symbols, DNS data of Spalart (1988); dashed lines, Rotta's model, Eq. (11.167); solid lines, Eq. (11.169).

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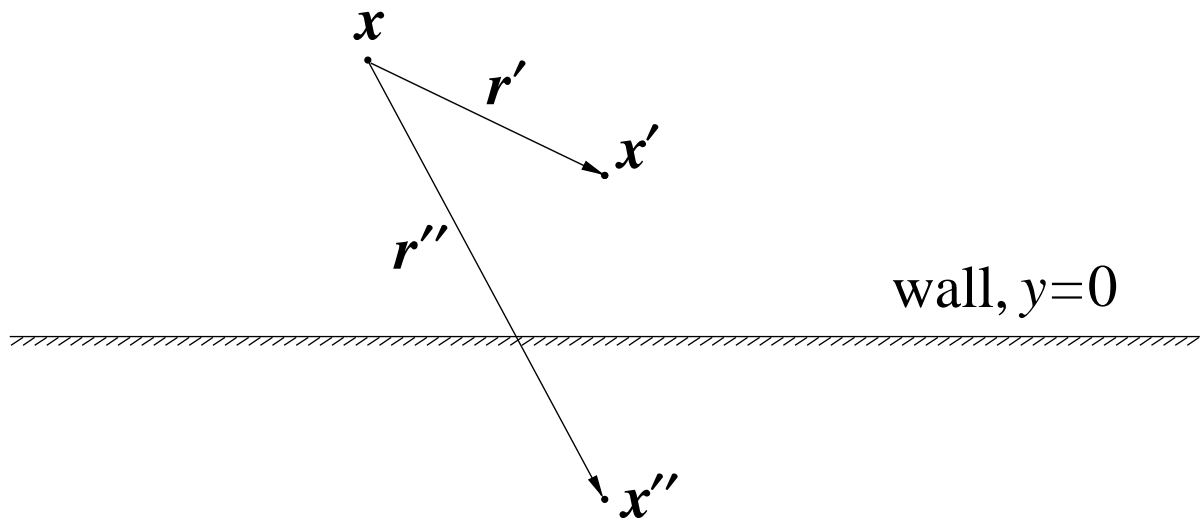


Figure 11.19: Sketch of the point  $\mathbf{x}'$  and its image  $\mathbf{x}''$ , showing the vectors  $\mathbf{r}'$  and  $\mathbf{r}''$  that appear in the Green's function solutions, Eqs. (11.181) and (11.182).

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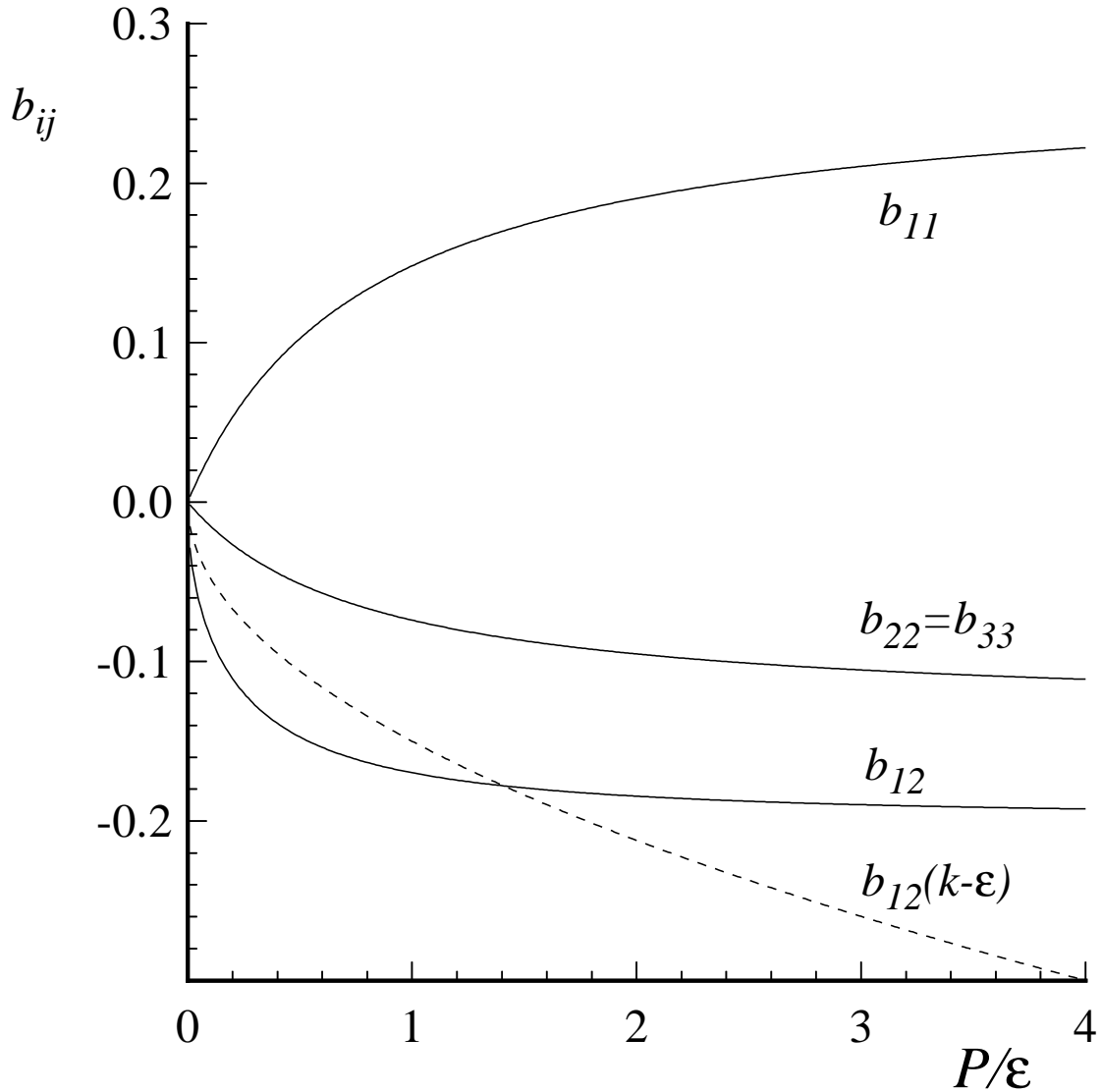


Figure 11.20: Reynolds-stress anisotropies as functions of  $\mathcal{P}/\epsilon$  according to the LRR-IP algebraic stress model. The dashed line shows  $b_{12}$  according to the  $k-\epsilon$  model.

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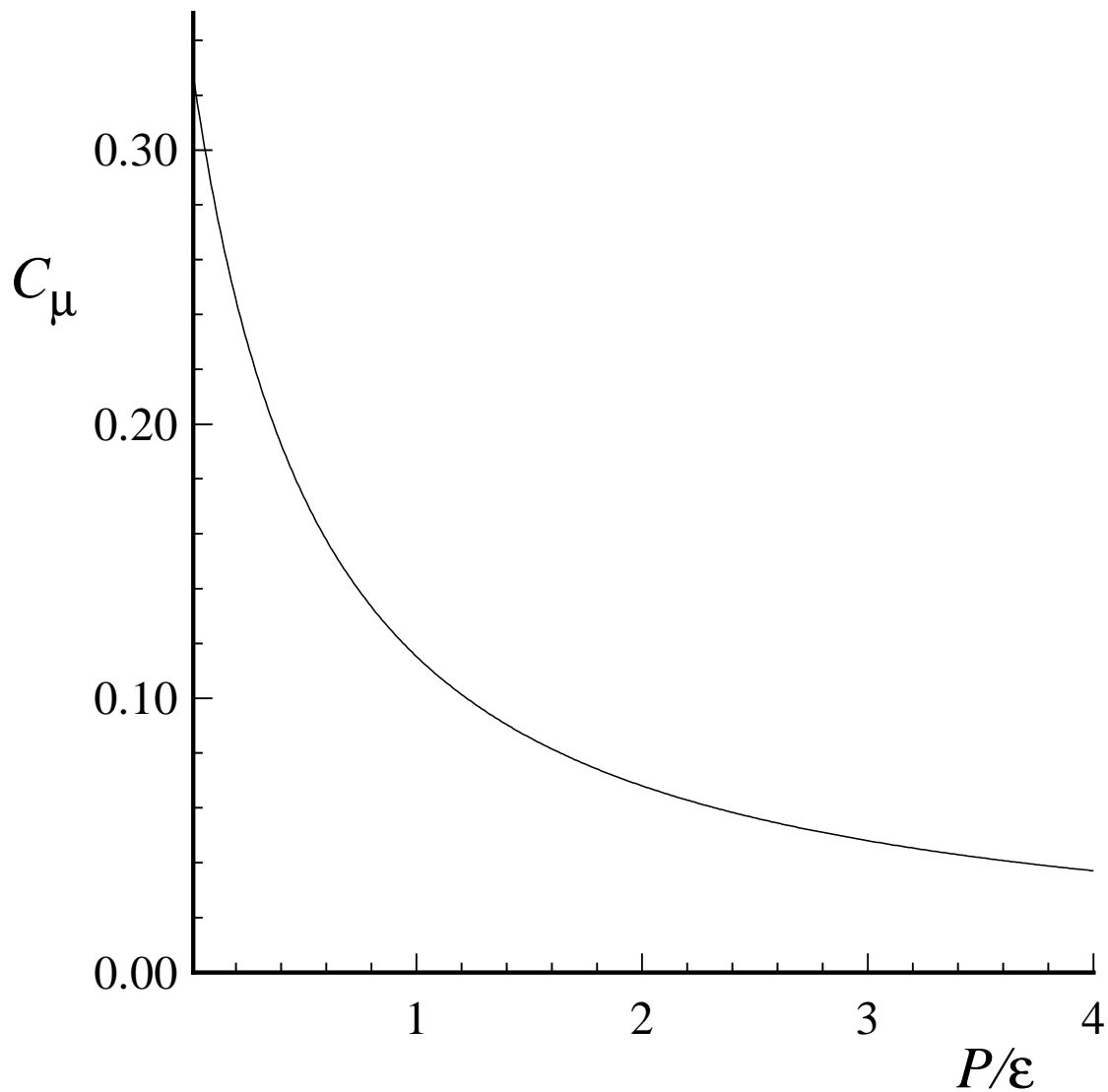


Figure 11.21: The value of  $C_\mu$  as a function of  $\mathcal{P}/\epsilon$  given by the LRR-IP algebraic stress model (Eq. 11.220).

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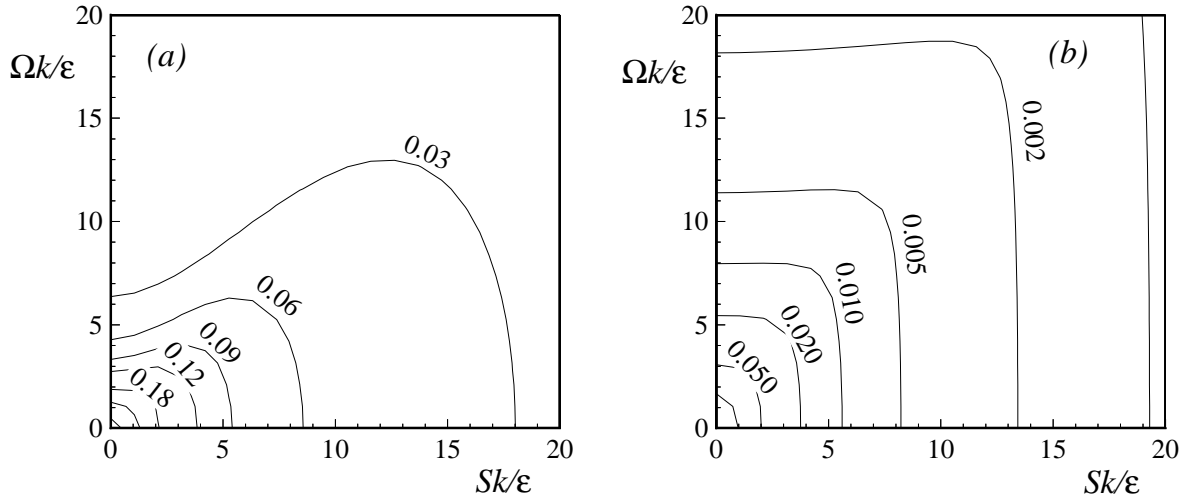


Figure 11.22: Contour plots of (a)  $C_\mu = -G^{(1)}$ , and (b)  $-G^{(2)}$ , for the LRR-IP nonlinear viscosity model (Eqs. 11.230–11.232).