Turbulent Flows

Stephen B. Pope

Cambridge University Press, 2000

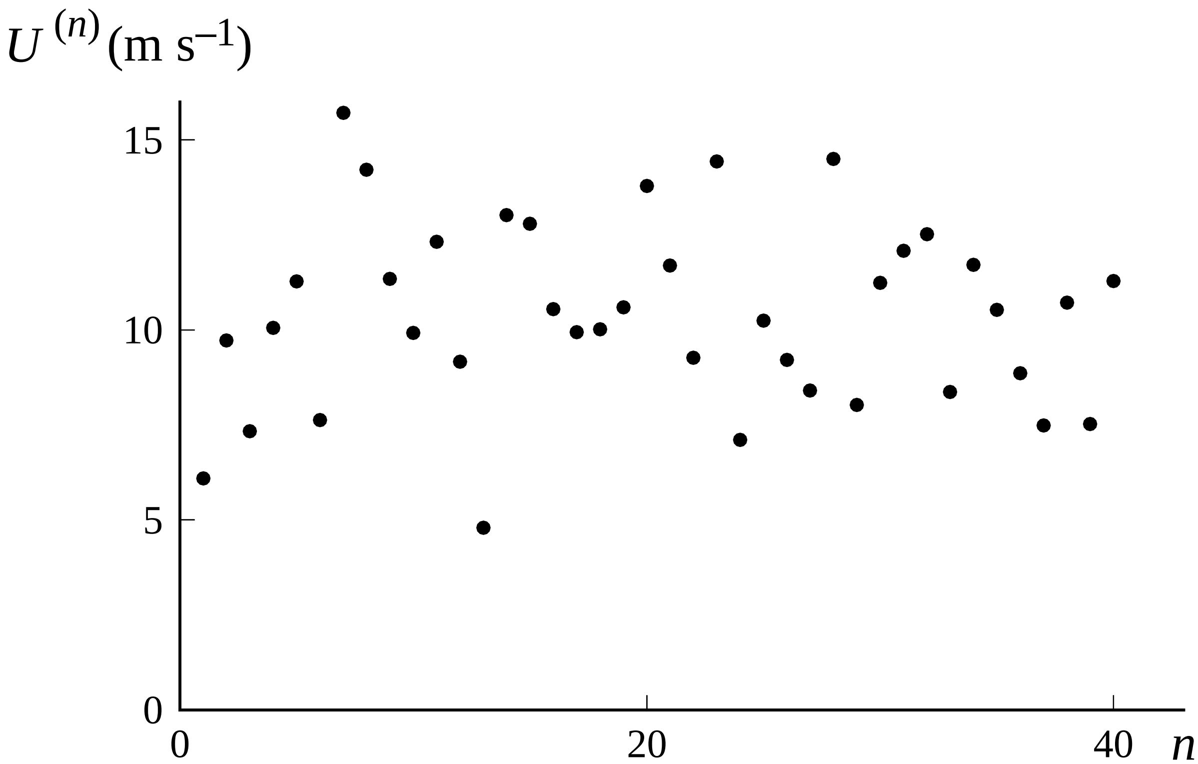

Figure 3.1: Sketch of the value $U^{(n)}$ of the random velocity variable U on the nth repetition of a turbulent flow experiment.

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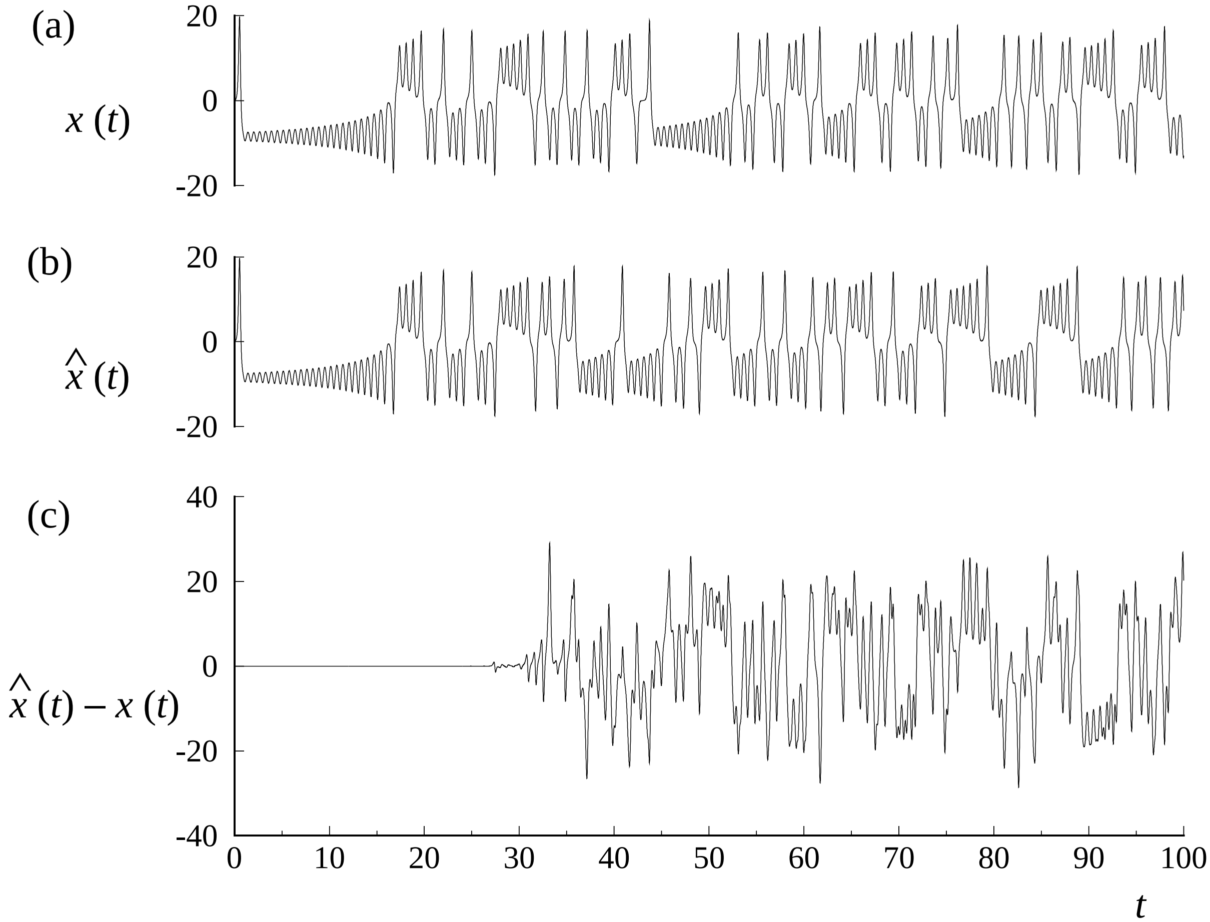

Figure 3.2: Time histories from the Lorenz equations (Eq. (3.1)): (a) $x(t)$ from the initial condition Eq. (3.2) (b) $\hat{x}(t)$ from the slightly different initial condition Eq. (3.3) (c) the difference $\hat{x}(t) - x(t)$.

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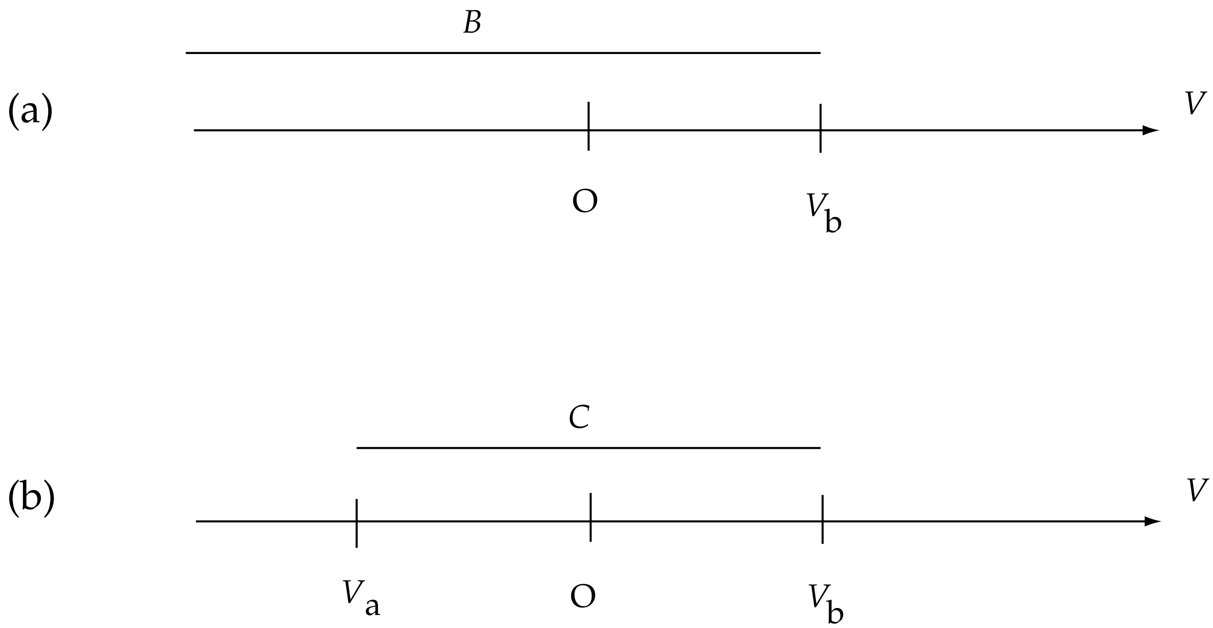

Figure 3.3: Sketch of the sample space of U showing the regions to the events (a) $B \equiv \{U < V_b\}$, and (b) $C \equiv \{V_a \leq U < V_b\}$.

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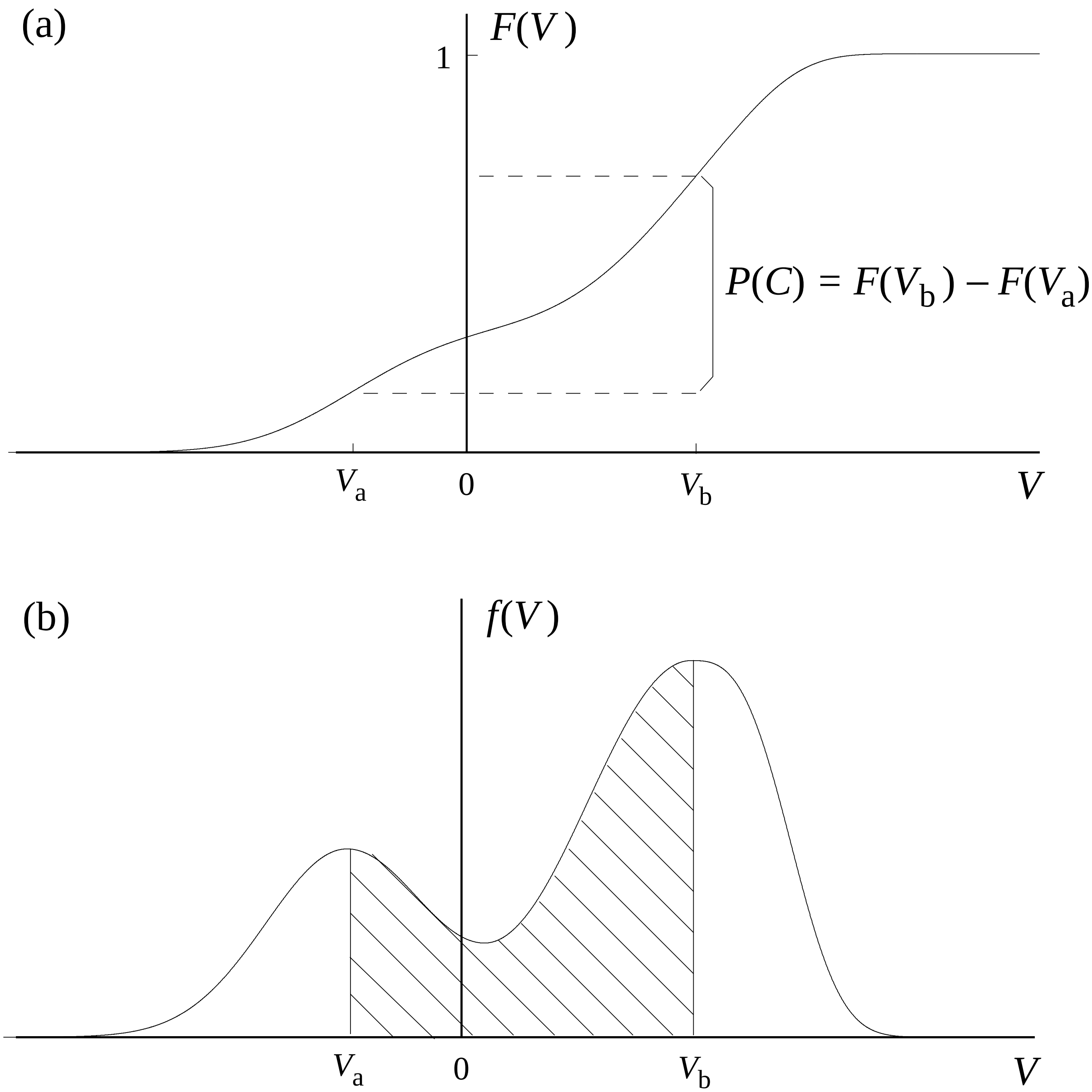

Figure 3.4: Sketch of (a) the CDF of the random variable U showing the probability of the event $C \equiv \{V_a \leq U < V_b\}$, and (b) the corresponding PDF. The shaded area in (b) is the probability of C.

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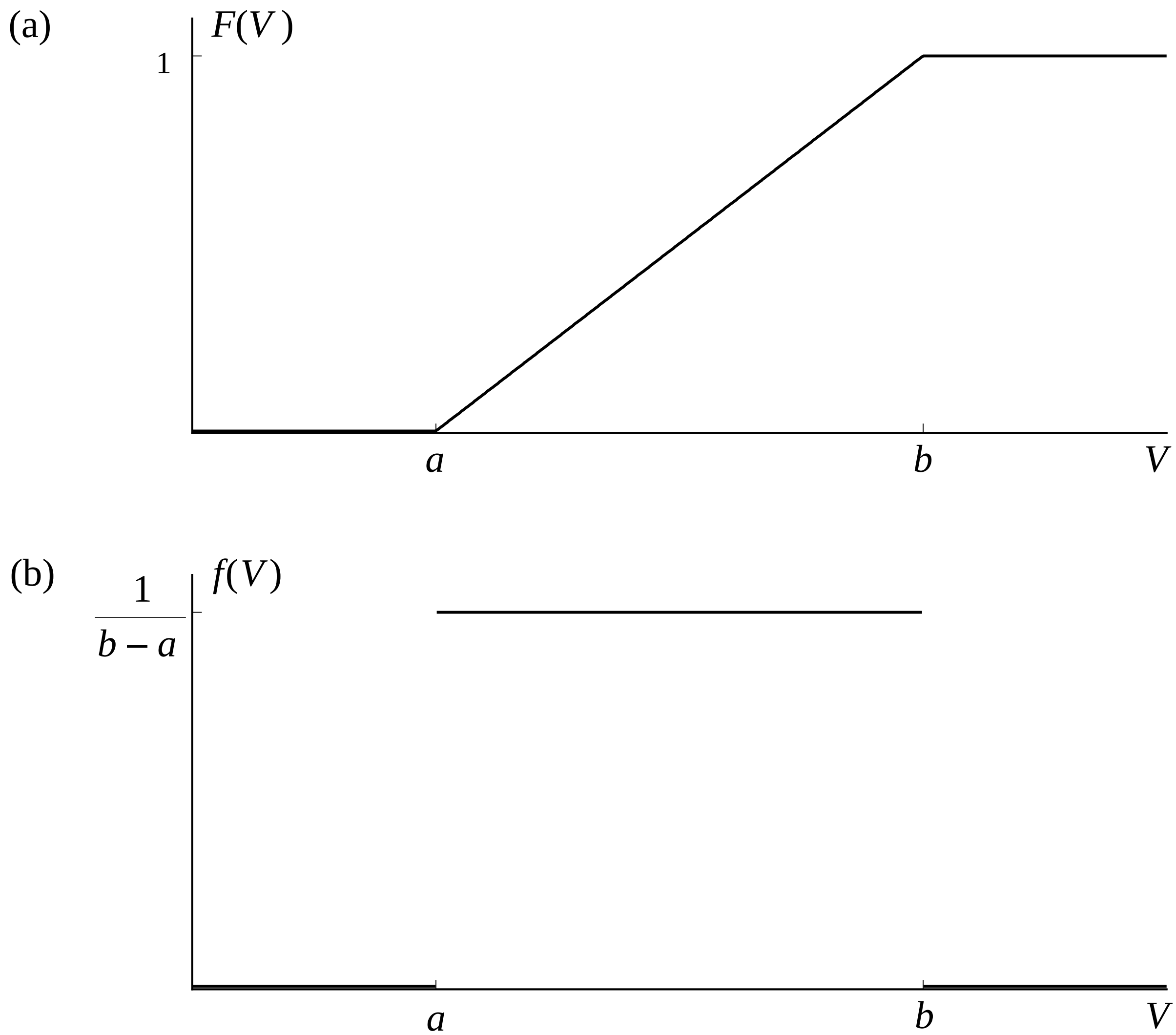

Figure 3.5: The CDF (a) and the PDF (b) of a uniform random variable (Eq. (3.39)).

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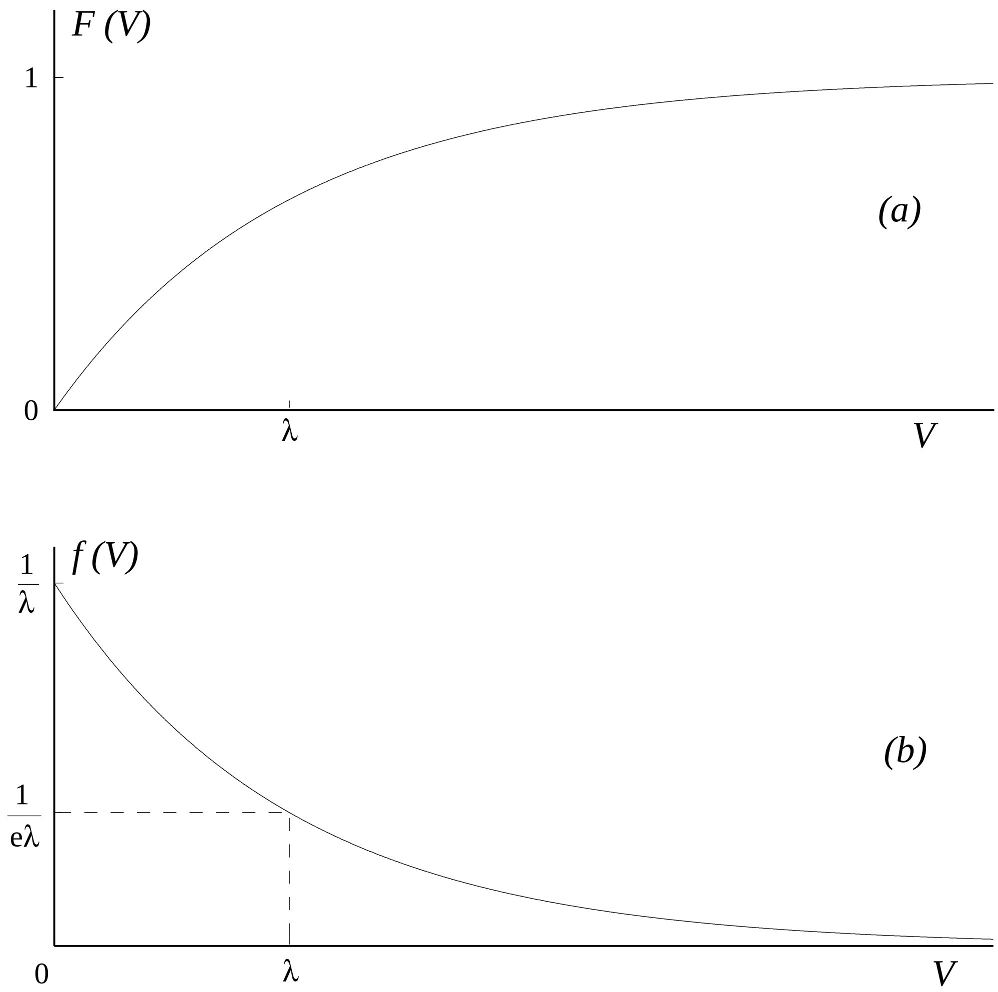

Figure 3.6: The CDF (a) and PDF (b) of an exponentially-distributed random variable (Eq. (3.40)).

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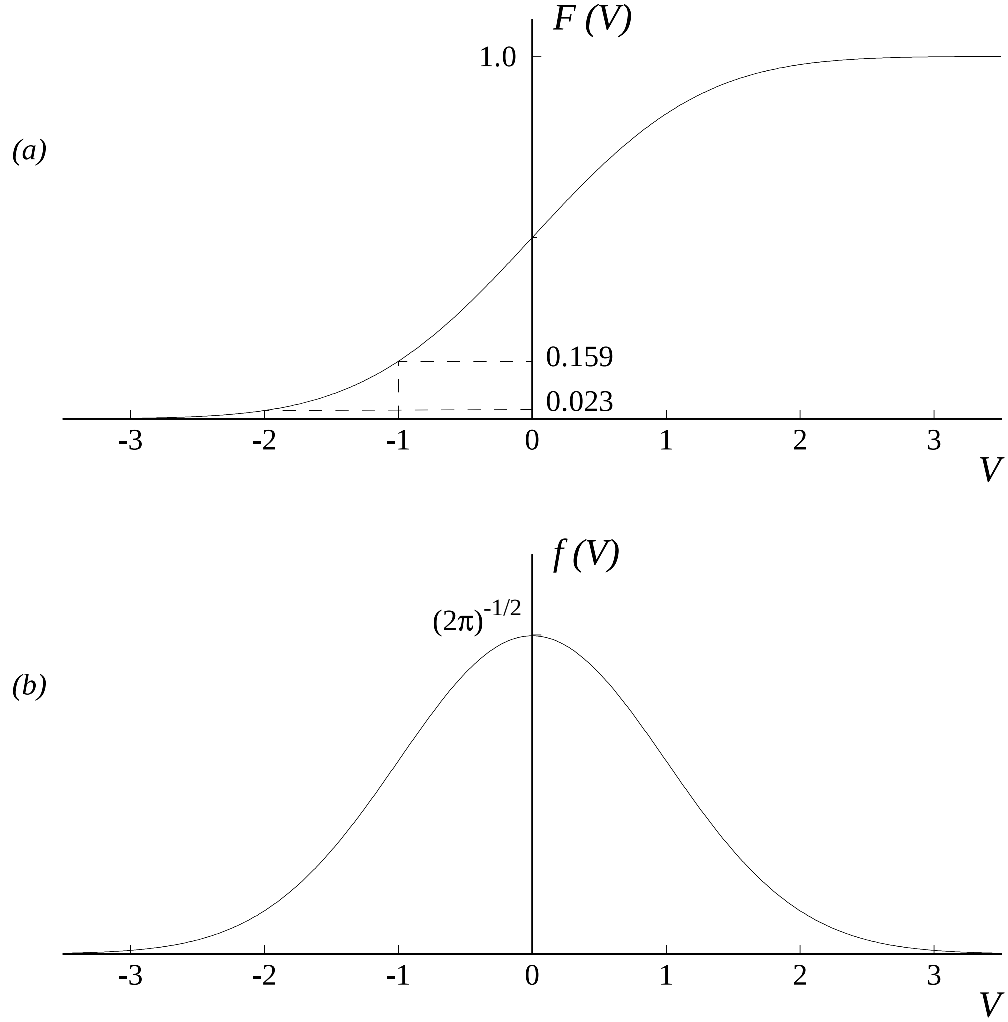

Figure 3.7: The CDF (a) and PDF (b) of a standardized Gaussian random variable.

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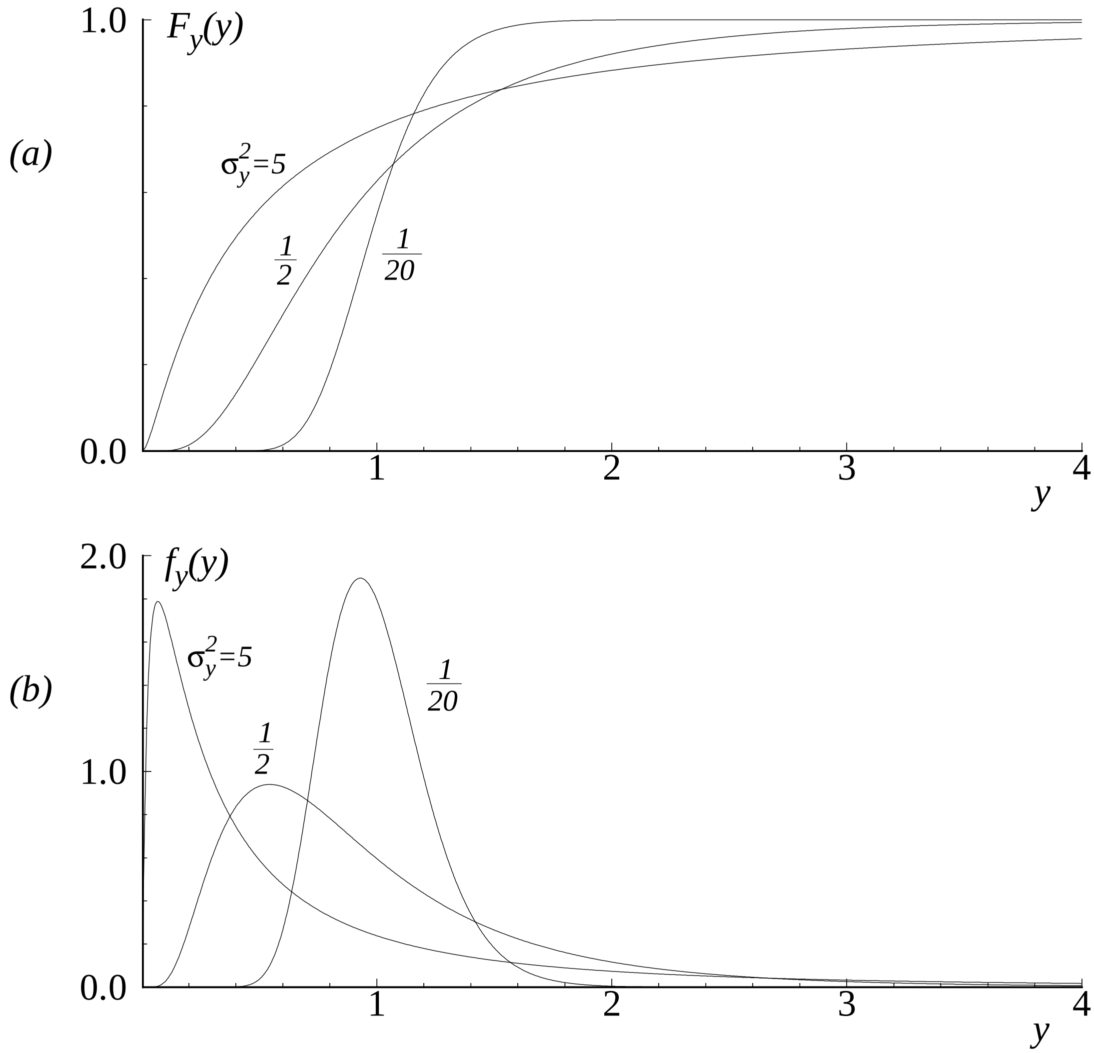

Figure 3.8: The CDF (a) and PDF (b) of the lognormal random variable Y with $\langle Y \rangle = 1$ and $\text{var}(Y) = \frac{1}{20}$, $\frac{1}{2}$ and 5.

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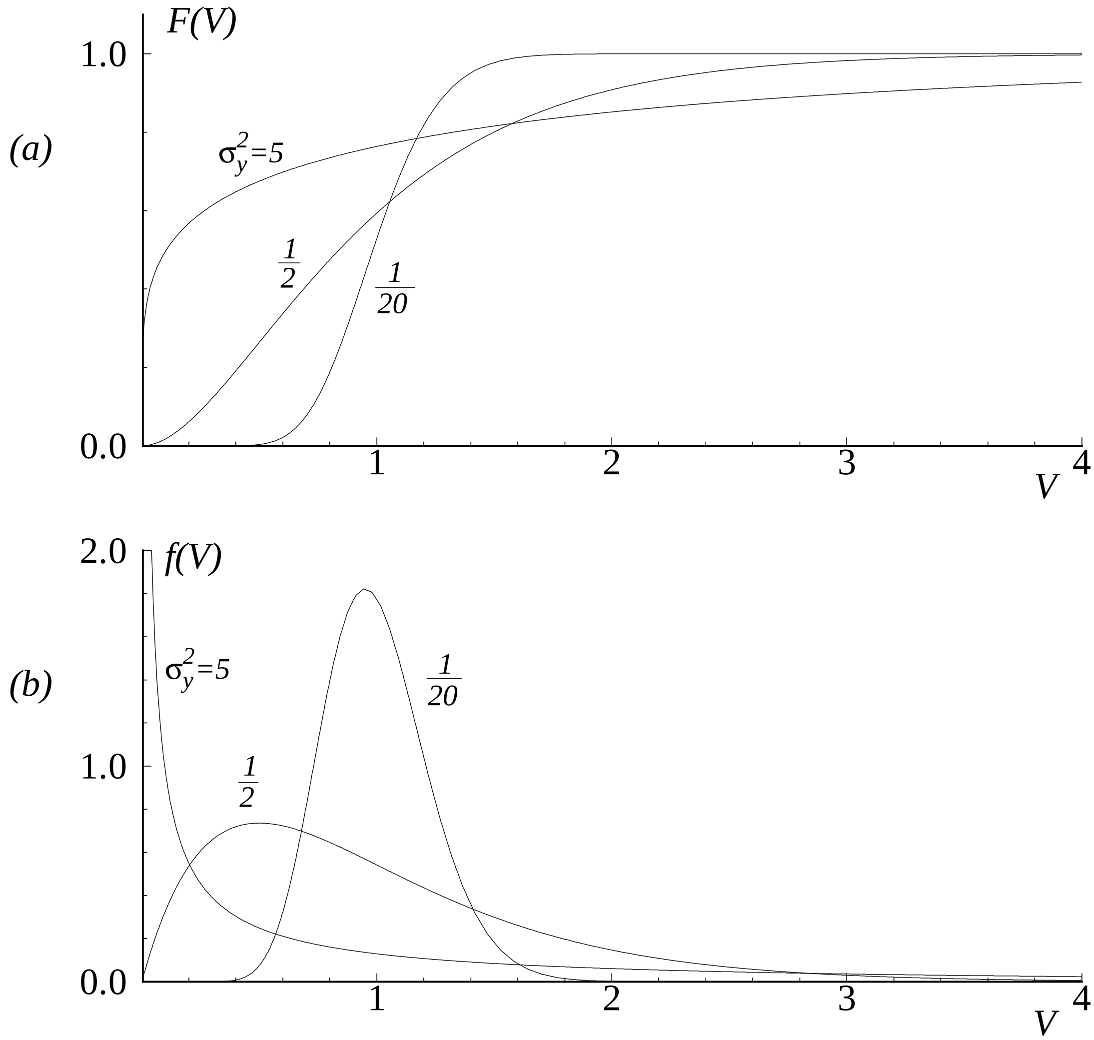

Figure 3.9: The CDF (a) and PDF (b) for the gamma distribution with mean $\mu = 1$ and variance $\sigma^2 = \frac{1}{20}$, $\frac{1}{2}$ and 5.

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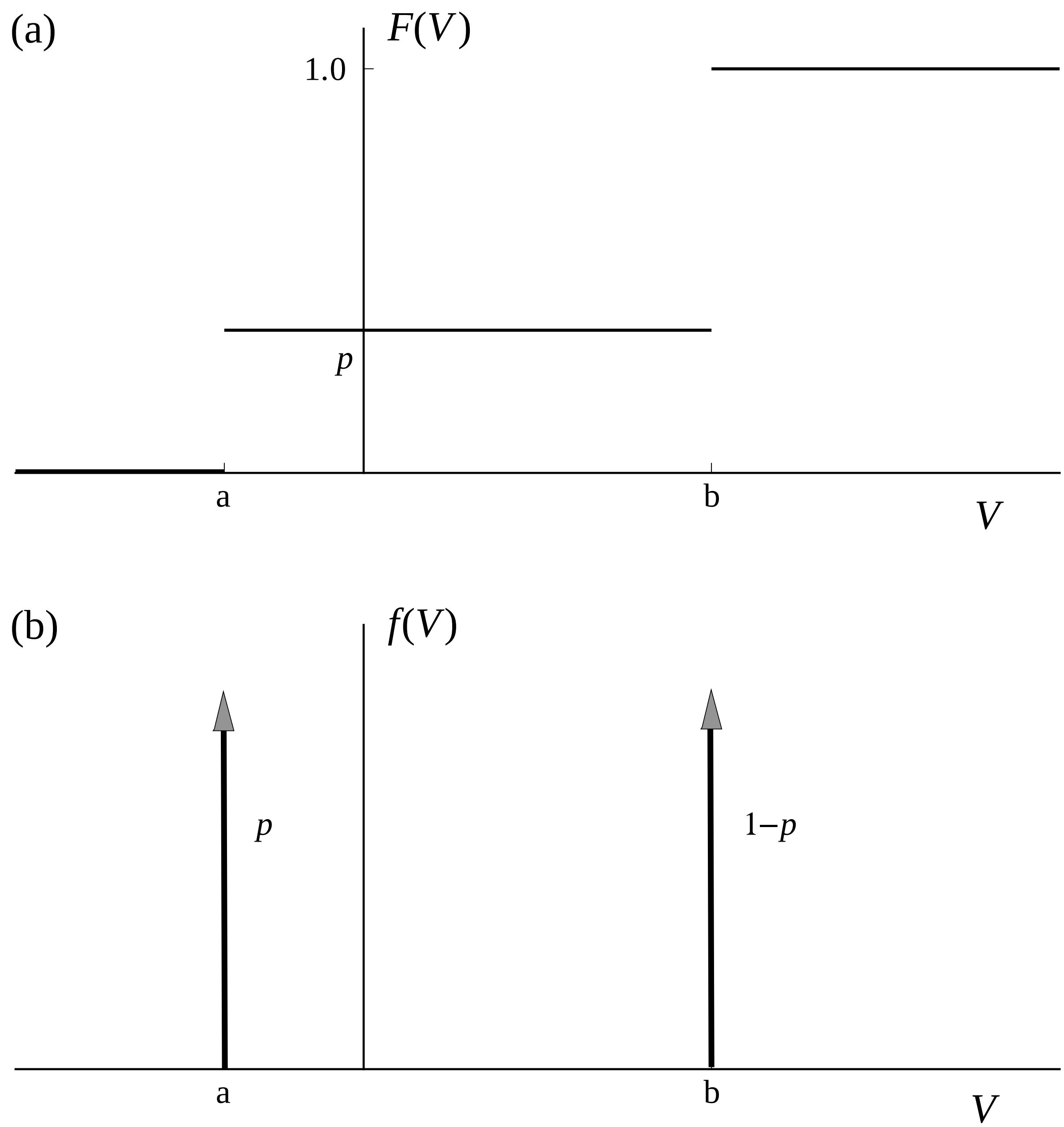

Figure 3.10: The CDF (a) and the PDF (b) of the discrete random variable U, Eq. (3.69).

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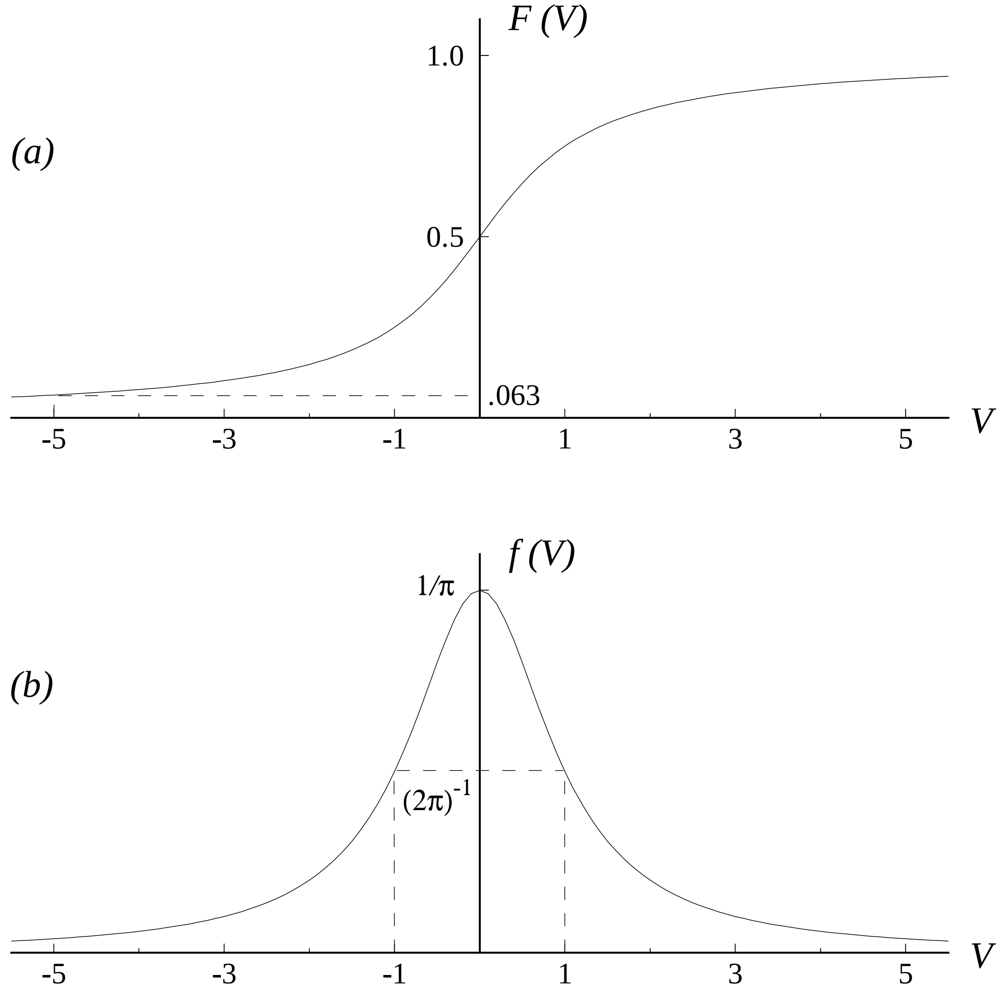

Figure 3.11: The CDF (a) and PDF (b) for the Cauchy distribution (Eqs. (3.79) and (3.80)) with $c = 0,\ w = 1$.

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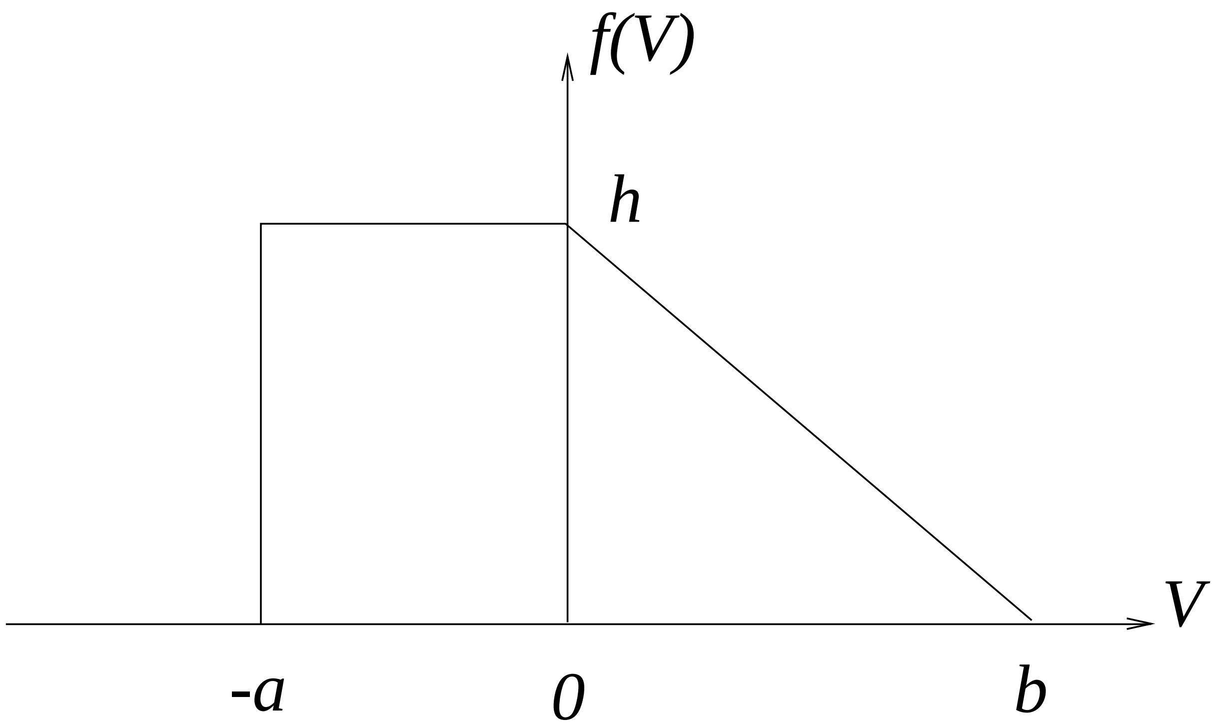

Figure 3.12: Sketch of the standardized PDF in Exercise 3.13.

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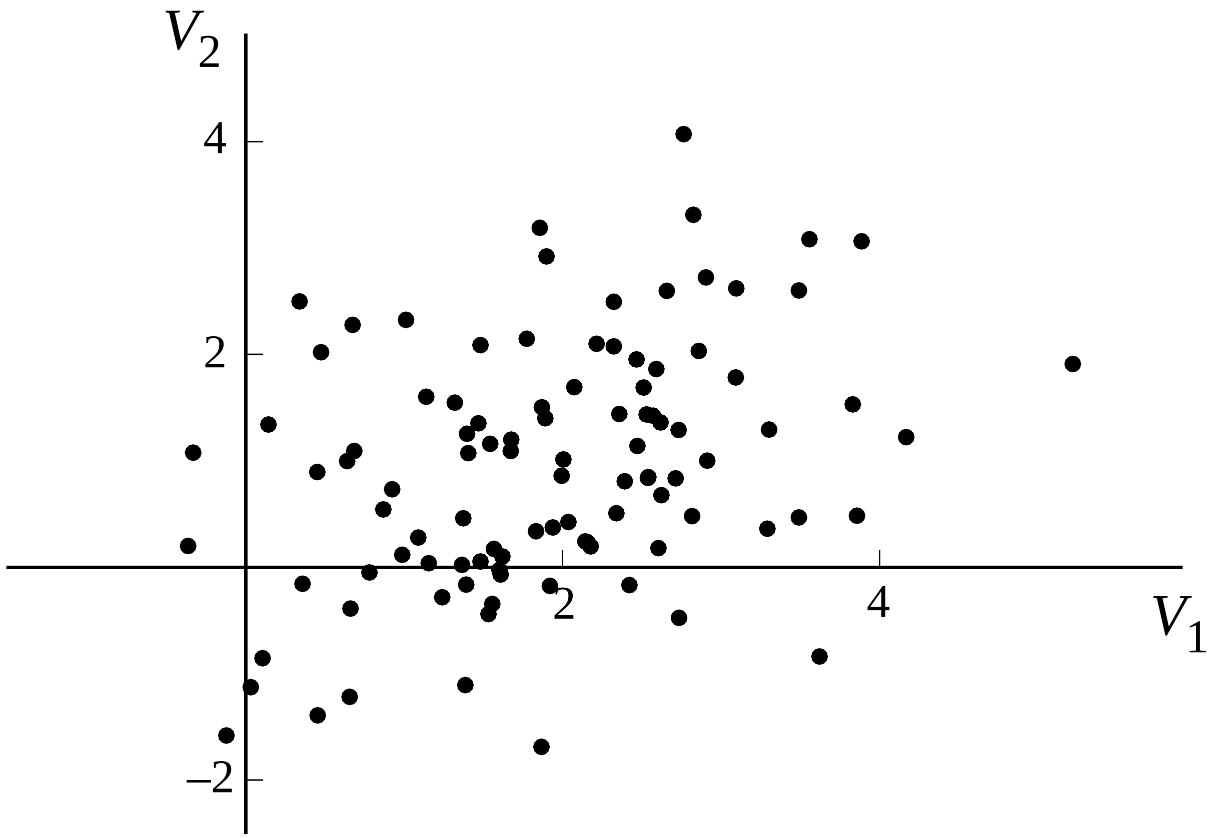

Figure 3.13: Scatter plot in the V_1-V_2 sample space of 100 samples of the joint random variables (U_1, U_2). (In this example U_1 and U_2 are jointly normal with $\langle U_1 \rangle = 2$, $\langle U_2 \rangle = 1$, $\langle u_1^2 \rangle = 1$, $\langle u_2^2 \rangle = \frac{5}{16}$, $\rho_{12} = 1/\sqrt{5}$.)

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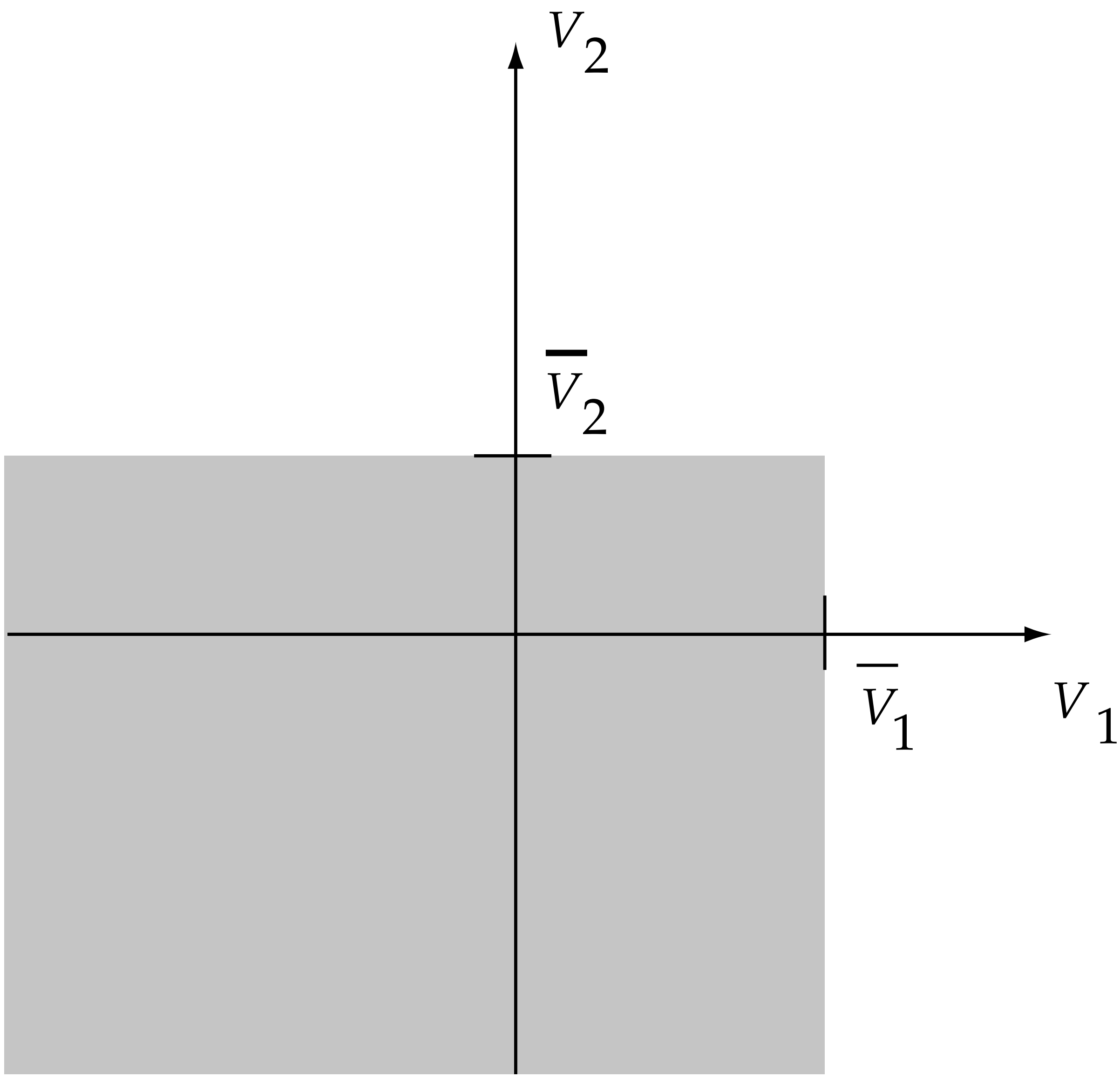

Figure 3.14: The V_1-V_2 sample space showing the region corresponding to the event $\{U_1 < \overline{V}_1,\ U_2 < \overline{V}_2\}$.

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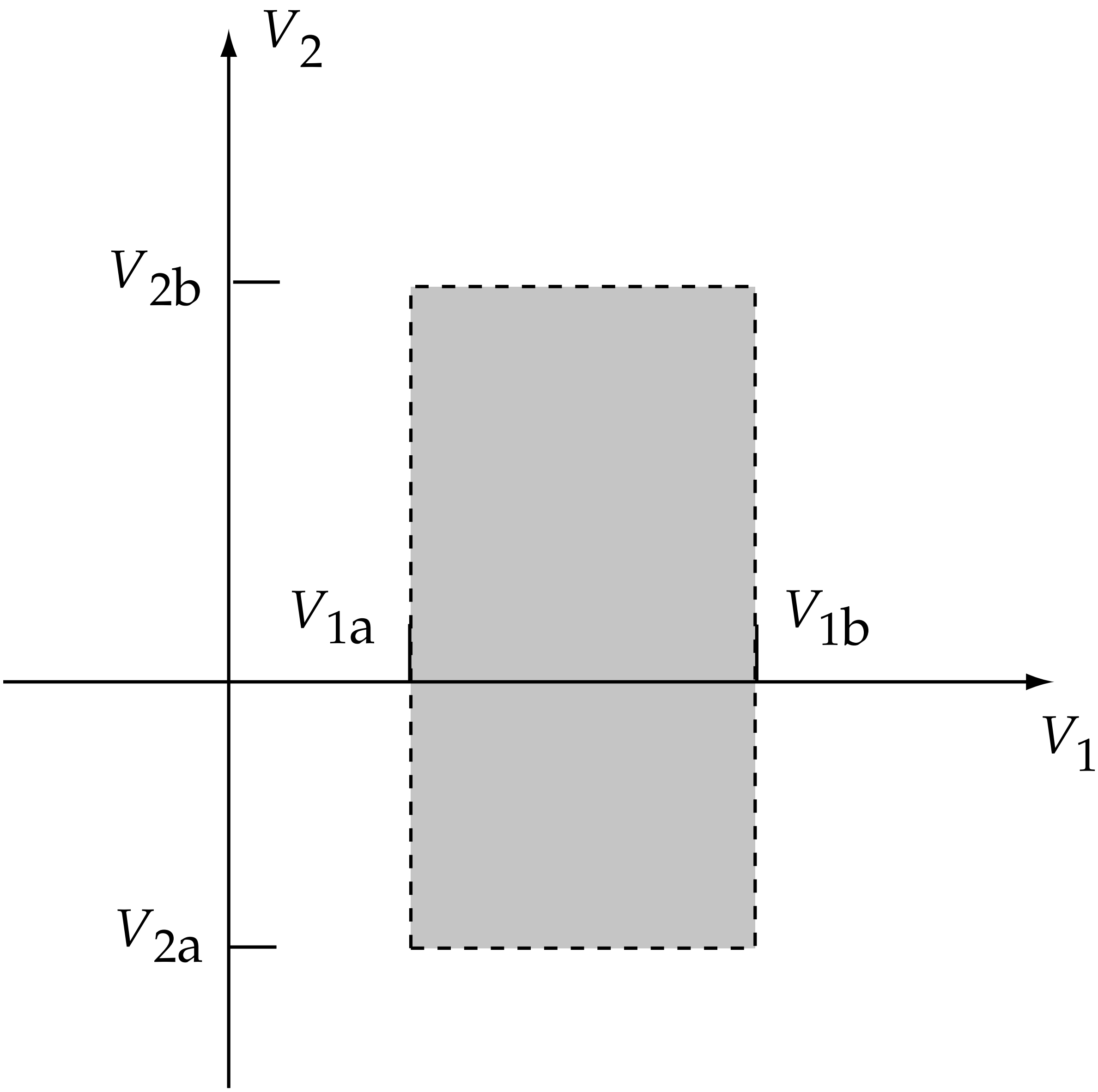

Figure 3.15: The V_1-V_2 sample space showing the region corresponding to the event $\{V_{1a} \leq U_1 < V_{1b},\ V_{2a} \leq U_2 < V_{2b}\}$, see Eq. (3.87).

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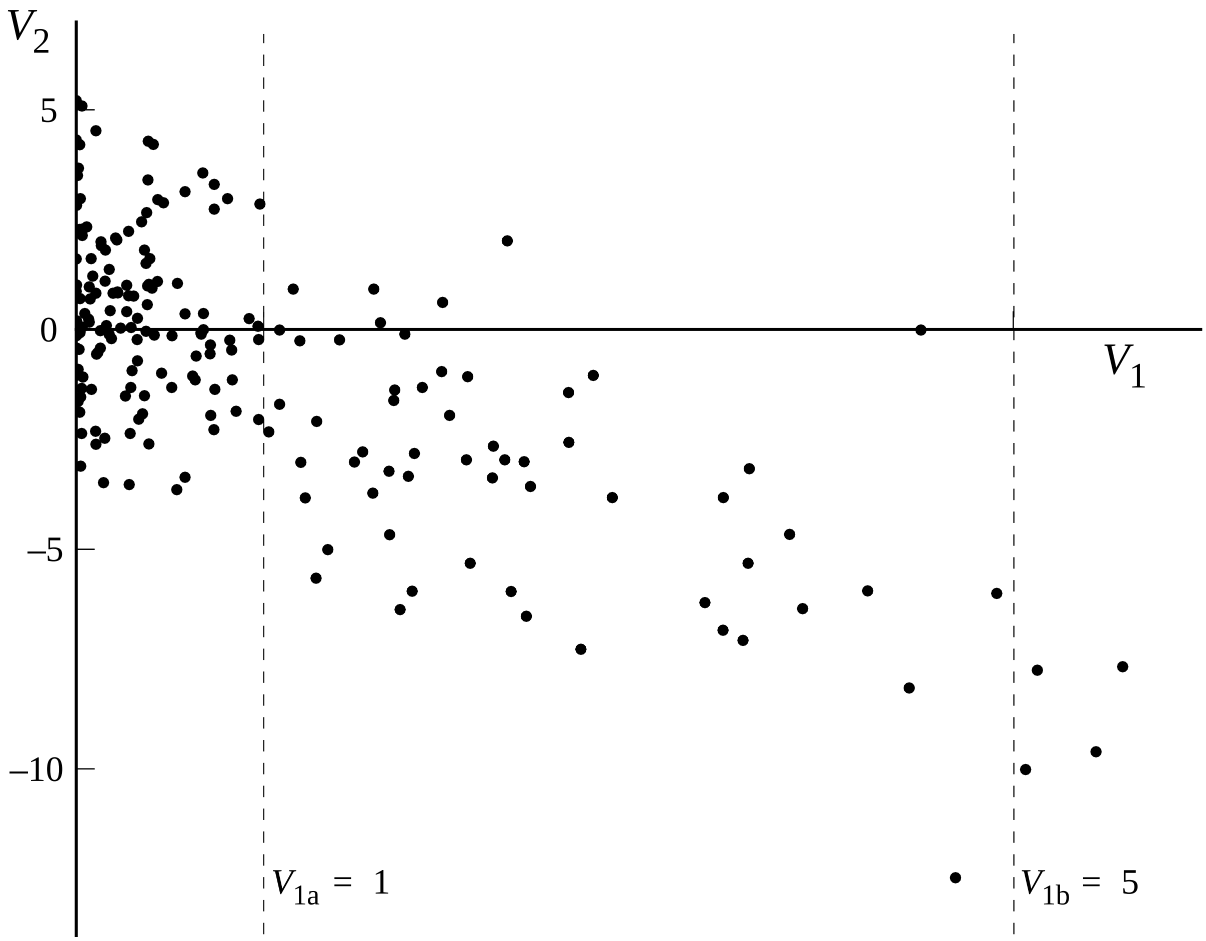

Figure 3.16: Scatter plot of negatively correlated random variables. $(\langle U_1 \rangle = 1,\ \langle U_2 \rangle = -1,\ \langle u_1^2 \rangle = 2,\ \langle u_2^2 \rangle = 12,\ \rho_{12} = -\sqrt{2/3})$.

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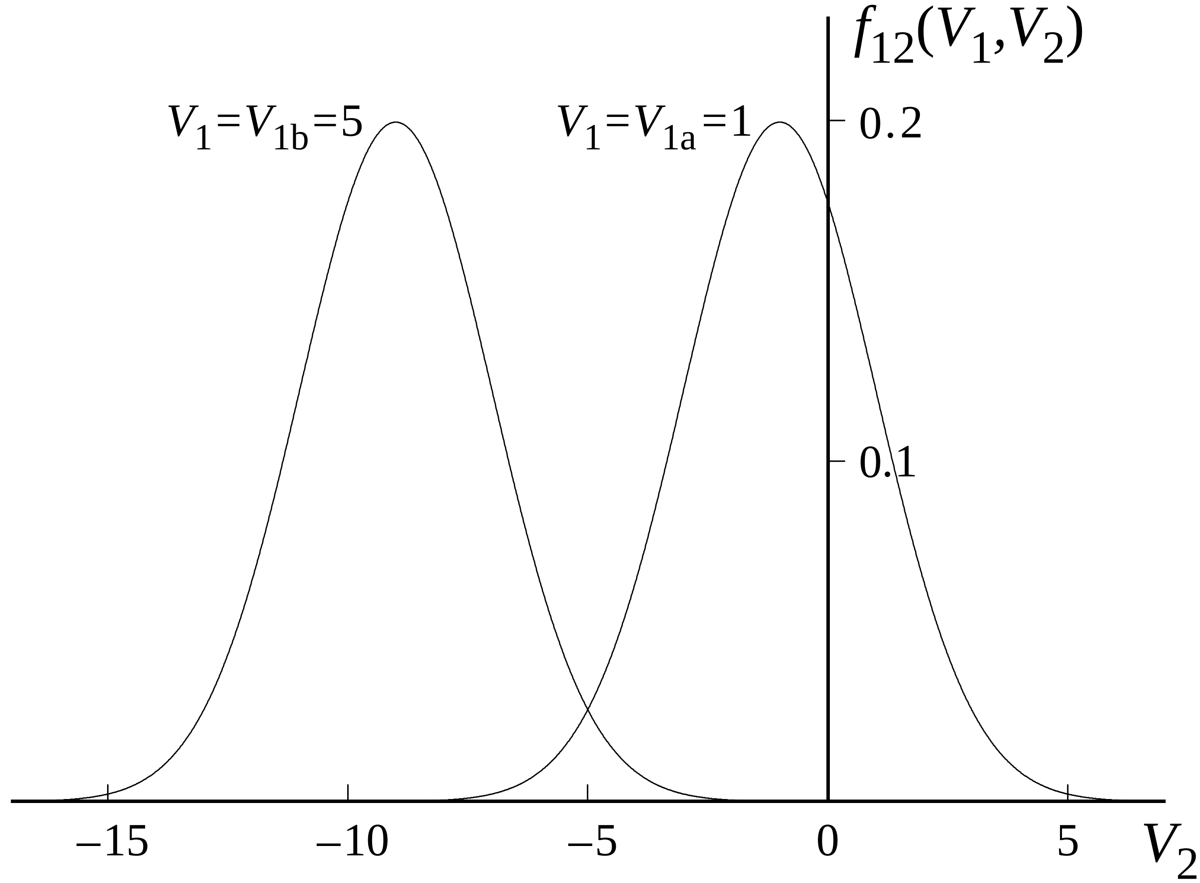

Figure 3.17: Joint PDF of the distribution shown in Fig. 3.16, plotted against V_2 for $V_1 = V_{1a} = 1$ and $V_1 = V_{1b} = 5$.

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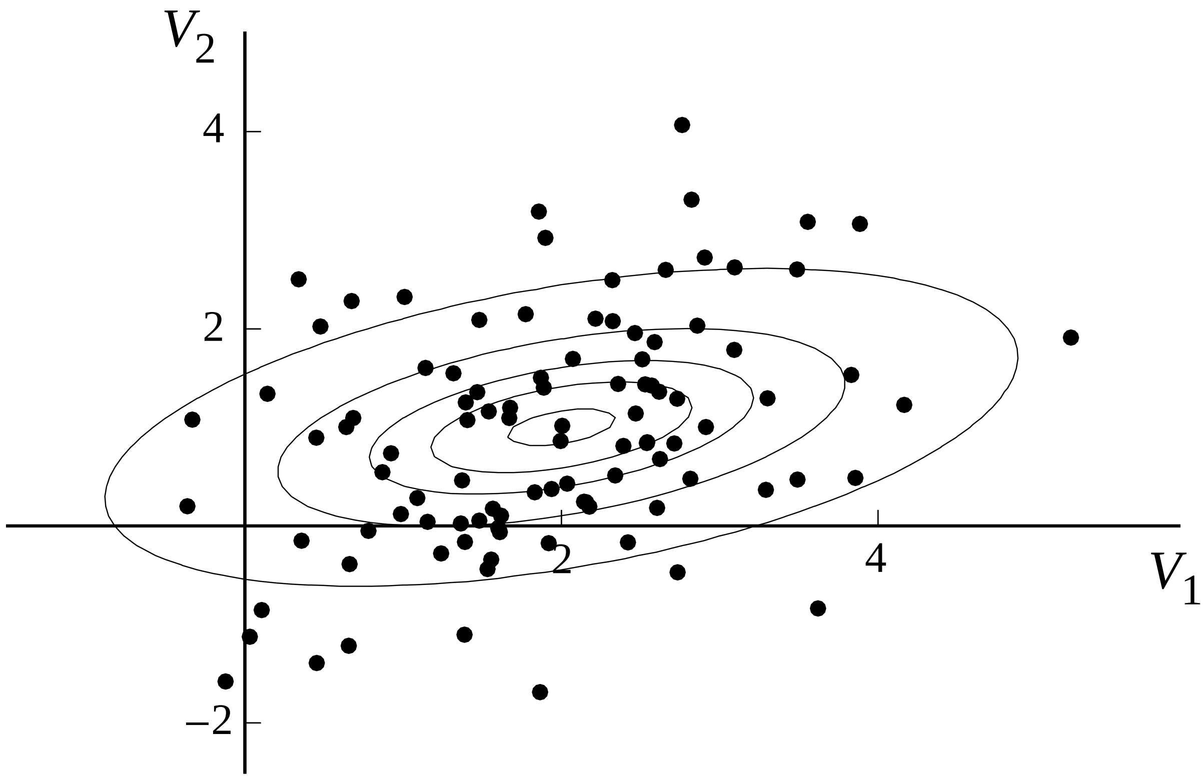

Figure 3.18: Scatter plot and constant probability density lines in the $V_1 - V_2$ plane for joint-normal random variables $(U_1,\ U_2)$ with $\langle U_1 \rangle = 2,\ \langle U_2 \rangle = 1,\ \langle u_1^2 \rangle = 1,\ \langle u_2^2 \rangle = \frac{5}{16},\ \rho_{12} = 1/\sqrt{5}$.

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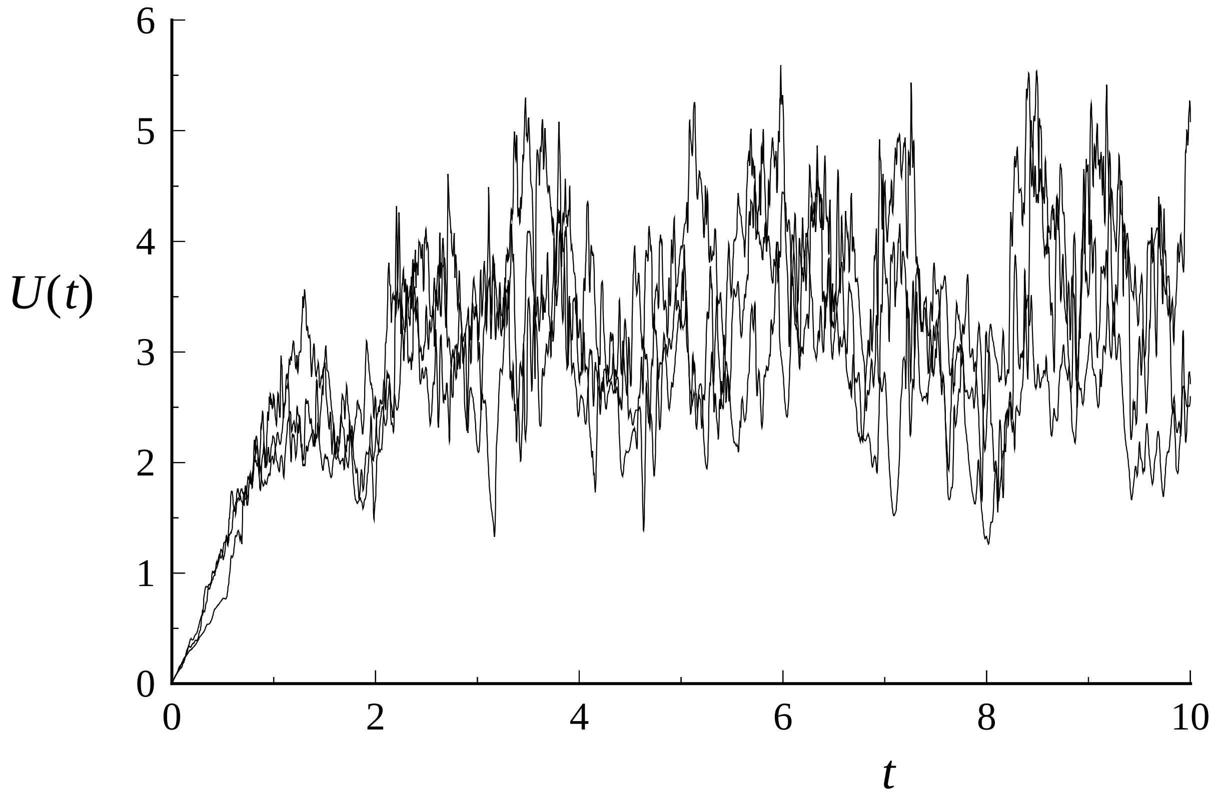

Figure 3.19: Sketch of sample paths of $U(t)$ from three repetitions of a turbulent flow experiment.

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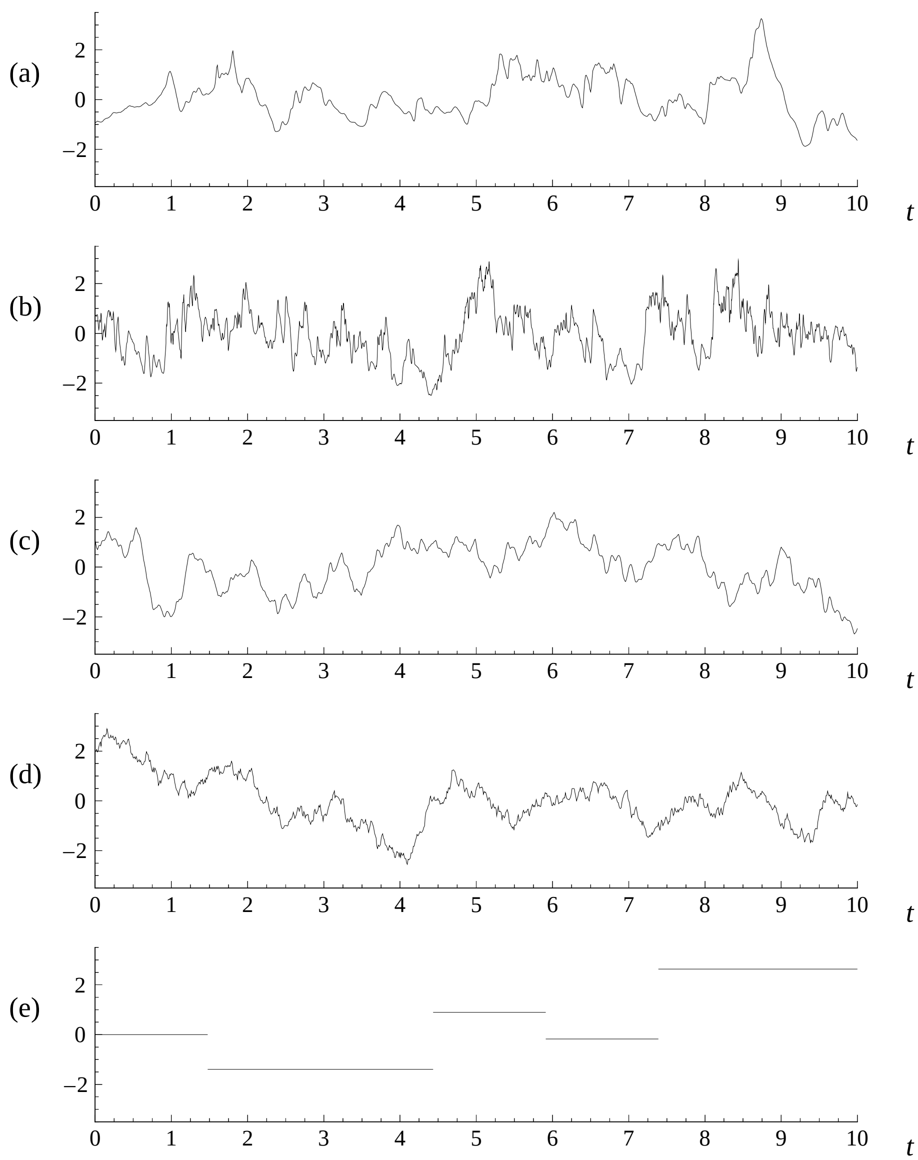

Figure 3.20: Sample paths of five statistically stationary random processes. The one-time PDF of each is a standardized Gaussian. (a) A measured turbulent velocity. (b) A measured turbulent velocity of a higher frequency than that of (a). (c) A Gaussian process with the same spectrum as that of (a). (d) An Ornstein-Uhlenbeck process (see Chapter 12) with the same integral timescale as that of (a). (e) A jump process with the same spectrum as that of (d).

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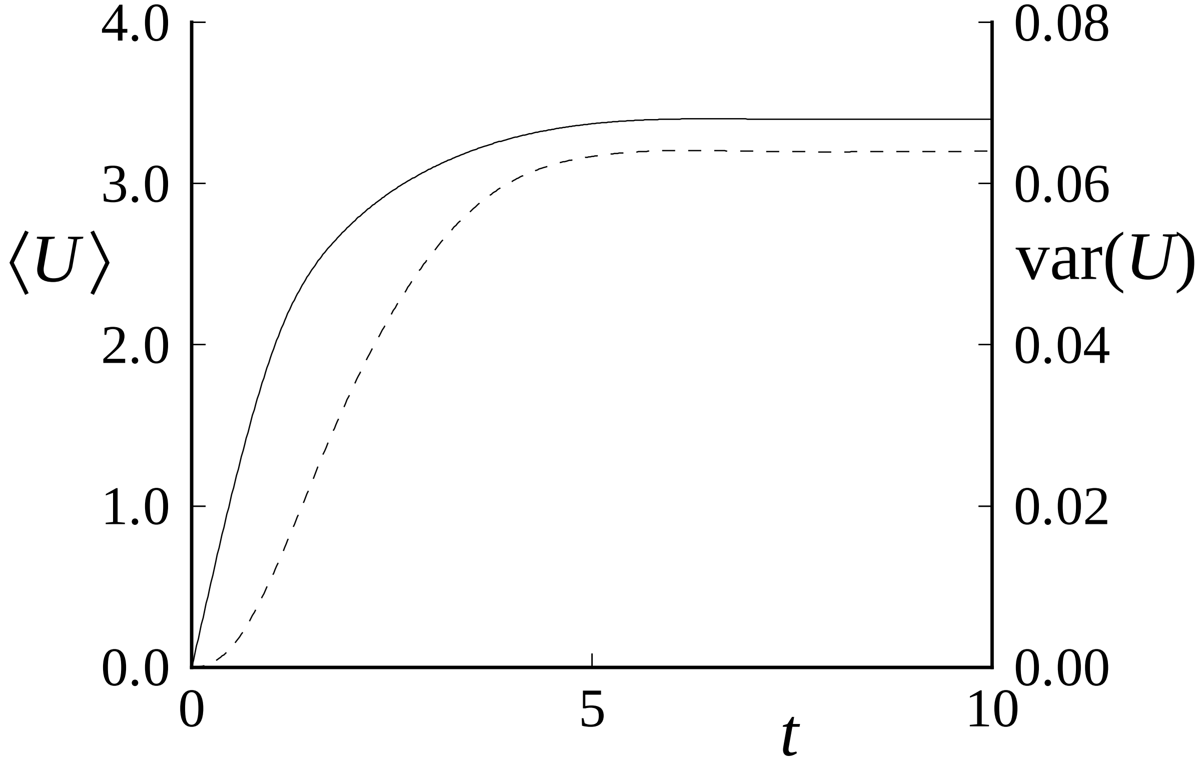

Figure 3.21: Mean $\langle U(t)\rangle$ (solid line) and variance $\langle u(t)^2\rangle$ of the process shown in Fig. 3.19.

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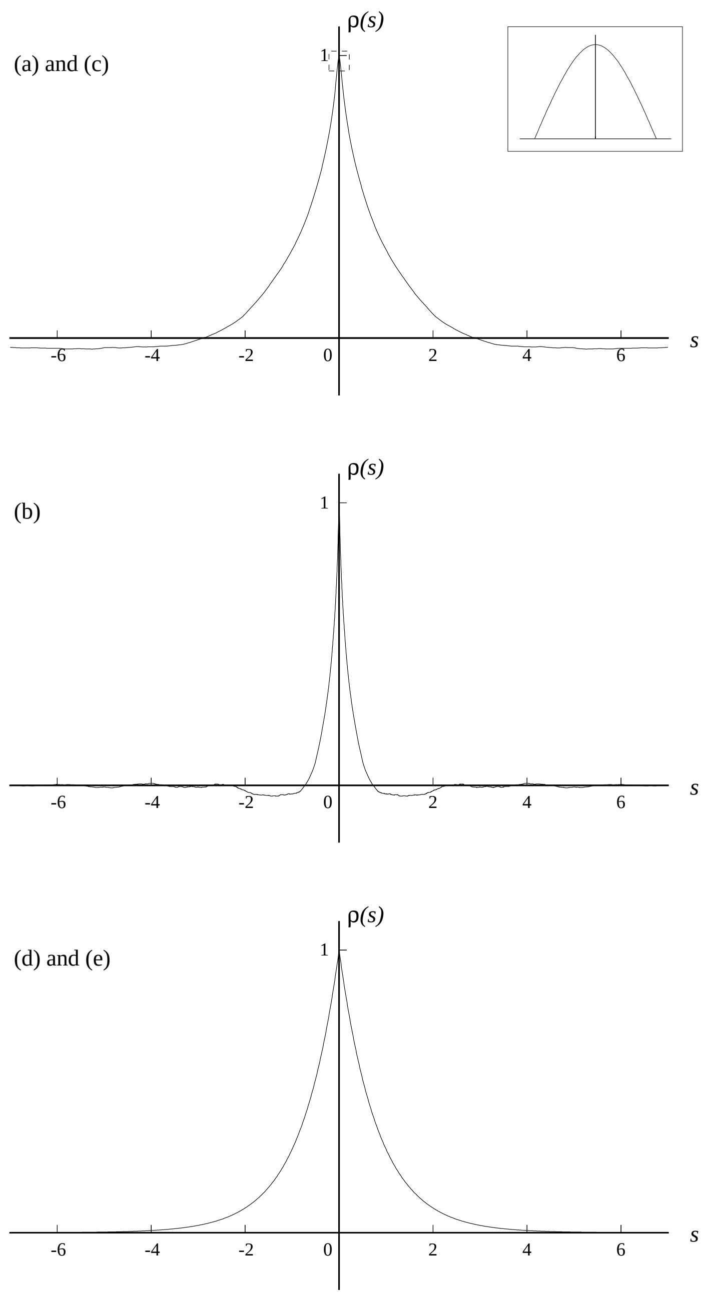

Figure 3.22: Autocorrelation functions of the processes shown on Fig. 3.20. As the inset shows, for processes (a) and (c) the autocorrelation function is smooth at the origin.

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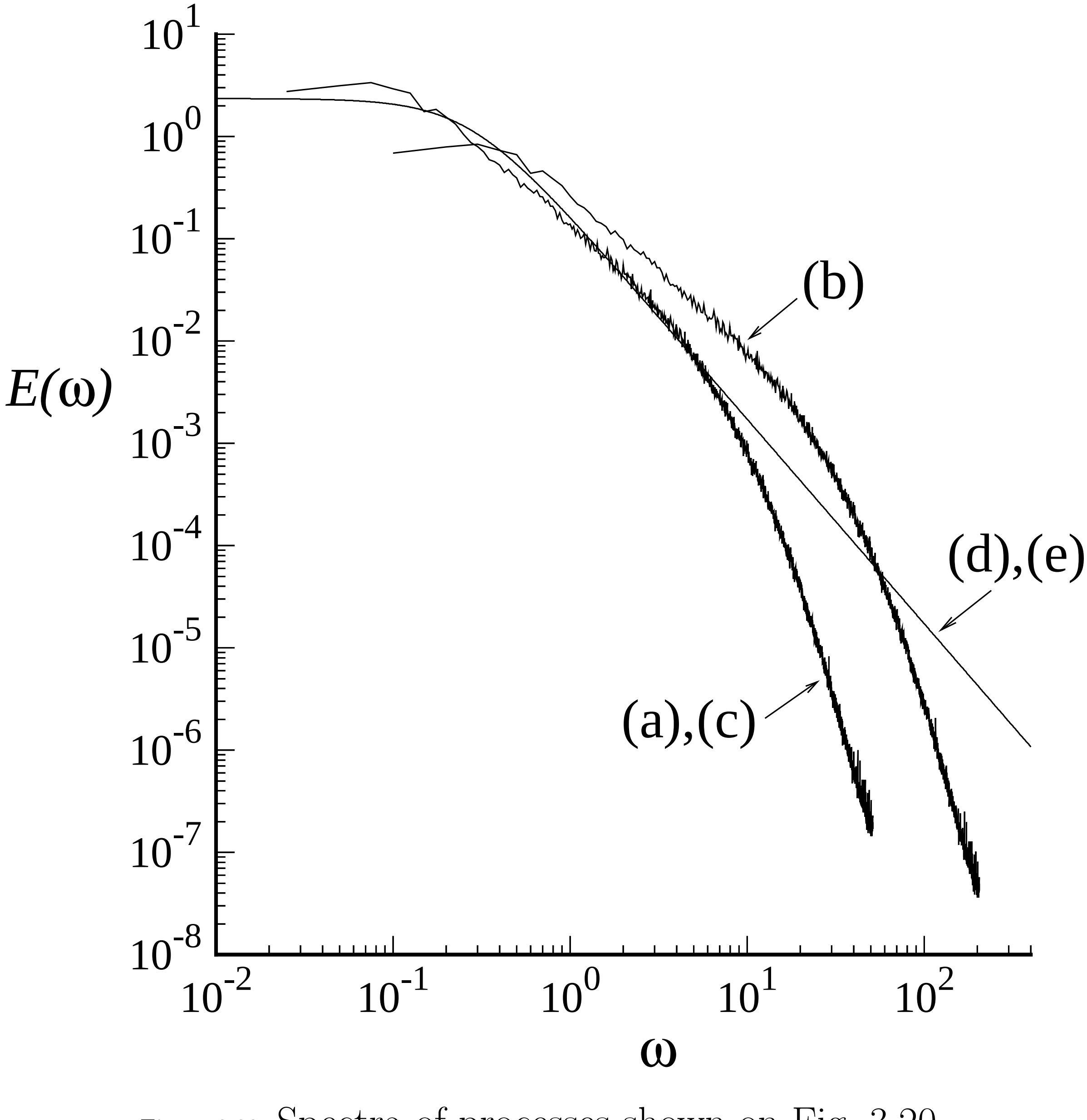

Figure 3.23: Spectra of processes shown on Fig. 3.20.

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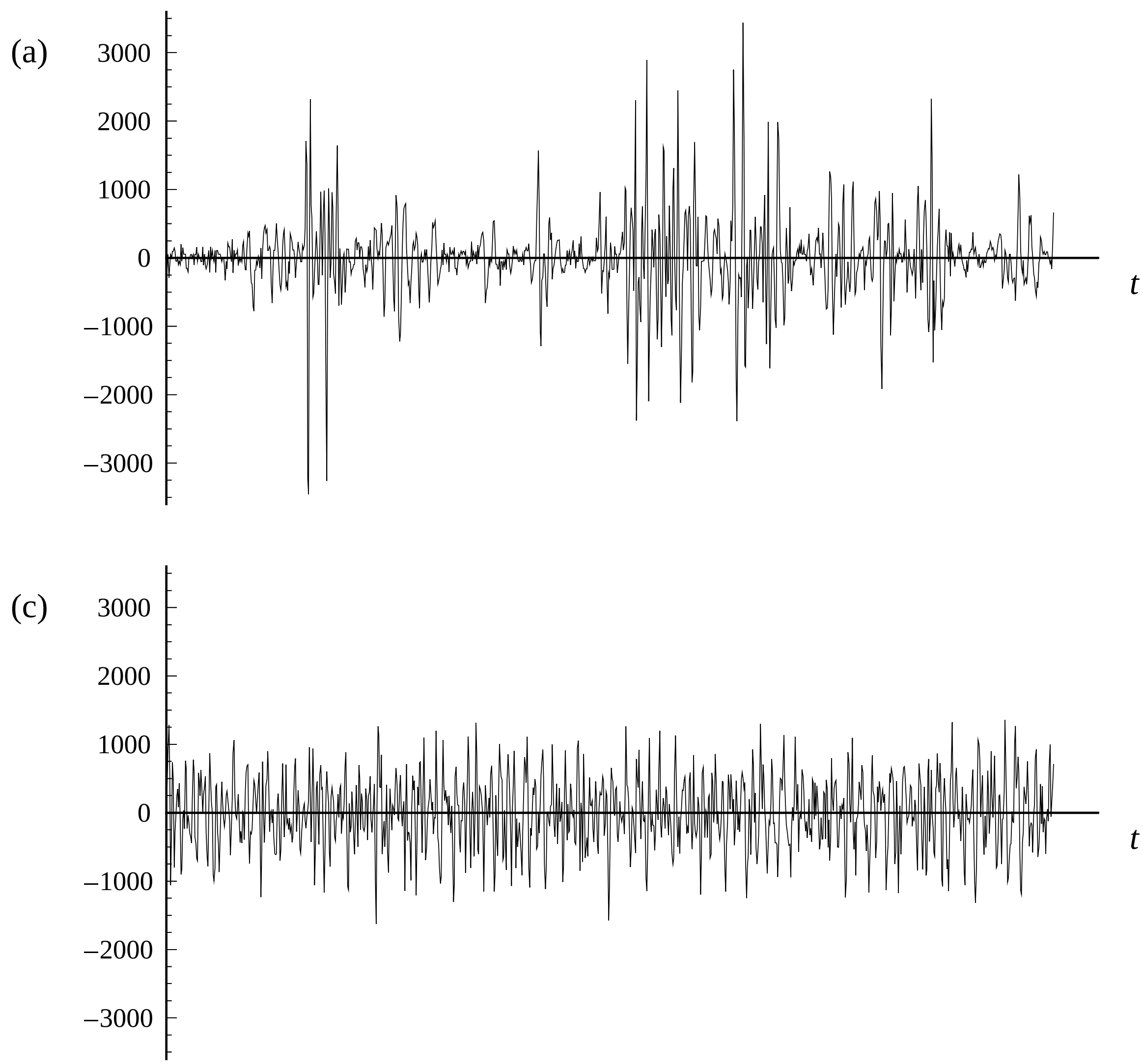

Figure 3.24: Sample paths of $\ddot{U}(t)$ for processes (a) and (c) shown on Fig. 3.20.

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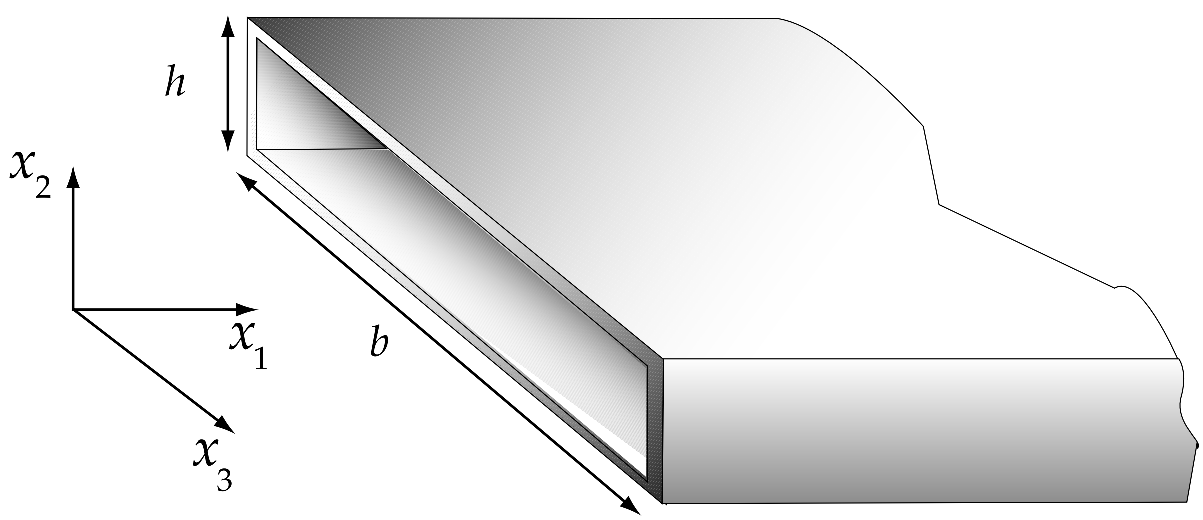

Figure 3.25: Sketch of a turbulent channel flow apparatus.

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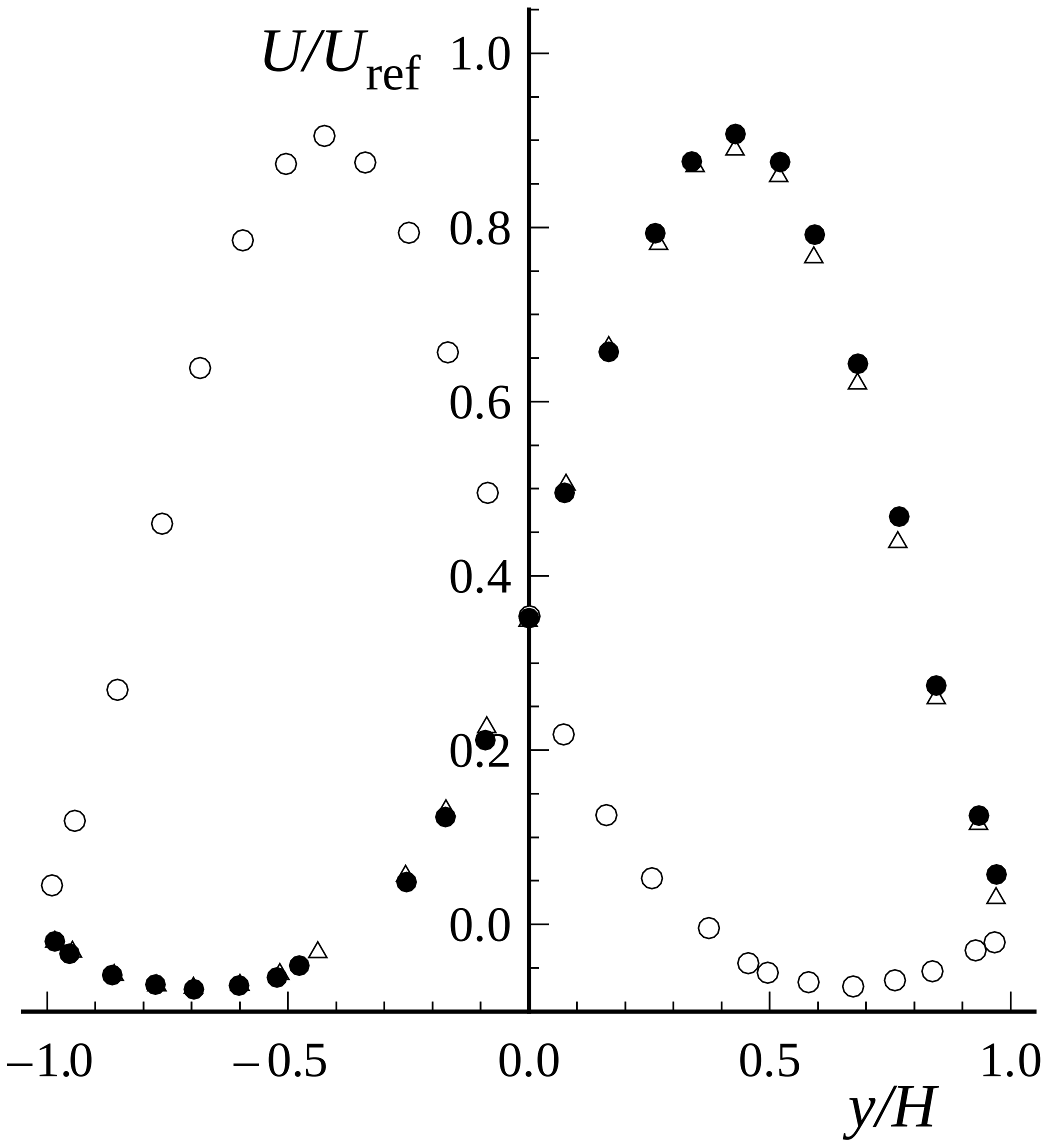

Figure 3.26: Velocity profiles measured by Durst et al. (1974) in the steady laminar flow downstream of a symmetric expansion in a rectangular duct. The geometry and boundary conditions are symmetric about the plane $y = 0$. Symbols: ○, stable state 1; △ stable state 2; •, reflection of profile 1 about the y axis.