Turbulent Flows
Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise A.12

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From Eq(A.61), the components of the vorticity are

$$\omega_i = \varepsilon_{ijk} \frac{\partial U_k}{\partial x_j} \tag{1}$$

Since ε_{ijk} is the alternating symbol, we have

$$\varepsilon_{ijk} = -\varepsilon_{ikj} \tag{2}$$

and therefore interchanging k and j in Eq.(A 61) we obtain

$$\omega_i = -\varepsilon_{ijk} \frac{\partial U_j}{\partial x_k} \tag{3}$$

Combining Eq.(1) and Eq.(3) we obtain

$$2\,\omega_i = \varepsilon_{ijk} \frac{\partial U_k}{\partial x_j} - \varepsilon_{ijk} \frac{\partial U_j}{\partial x_k} = -\varepsilon_{ijk} \left(\frac{\partial U_j}{\partial x_k} - \frac{\partial U_k}{\partial x_j} \right) \tag{4}$$

and finally

$$\omega_i = -\varepsilon_{ijk}\,\Omega_{jk} \tag{5}$$

where Ω_{jk} is the rate of rotation tensor defined by

$$\Omega_{jk} \equiv \frac{1}{2} \left(\frac{\partial U_j}{\partial x_k} - \frac{\partial U_k}{\partial x_j} \right) \tag{6}$$