# Exercise 3.25 Solution

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(a) Since $x_1$, $x_2$ and $x_3$ are independent variables, it follows that $\frac{\partial x_1}{\partial x_1} = 1$ and $\frac{\partial x_1}{\partial x_2} = 0$ etc. Therefore, by extending this reasoning we have

$$\begin{aligned} \frac{\partial x_i}{\partial x_j} =& 1 \qquad\qquad\qquad \textit{when } i \neq j \\ =& 0 \qquad\qquad\qquad \textit{when } i = j \end{aligned} \tag{1}$$

Therefore, we have

$$\frac{\partial x_i}{\partial x_j} = \delta_{ij} \tag{2}$$

(b) From Eq. (A.5), we have

$$\begin{aligned} \frac{\partial \mathbf{x}}{\partial x_i} =& \frac{\partial (\mathbf{e}_j x_j)}{\partial x_i} \\ =& \mathbf{e}_j \frac{\partial x_j}{\partial x_i} \\ =& \mathbf{e}_j \delta_{ij} = \mathbf{e}_i \end{aligned} \tag{3}$$

(c) From the definition of $r$, we have

$$\begin{aligned} r =& [(\mathbf{e}_i x_i) \cdot (\mathbf{e}_j x_j)]^{1/2} \\ =& [\mathbf{e}_i \cdot \mathbf{e}_j x_i x_j]^{1/2} \\ =& [\delta_{ij} x_i x_j]^{1/2} = [x_i x_i]^{1/2} \end{aligned} \tag{4}$$

Therefore, we have

$$\begin{aligned}\frac{\partial r}{\partial x_i} =& \frac{\partial (\mathbf{x}\cdot\mathbf{x})^{1/2}}{\partial x_i}\\ =& \frac{\partial (x_j x_j)^{1/2}}{\partial x_i}\\ =& \frac{1}{2}(x_j x_j)^{1/2} 2x_i\\ =& \frac{x_i}{(x_j x_j)^{1/2}} = \frac{x_i}{r}\end{aligned} \tag{5}$$

(d) From Eq. (A.17), we have

$$\bar{x}_j = a_{kj} x_k \tag{6}$$

Differentiating both sides, we get

$$\frac{\partial \bar{x}_j}{\partial x_i} = a_{kj}\frac{\partial x_k}{\partial x_i} = a_{kj}\delta_{ik} = a_{ij} \tag{7}$$

(e) Eq. (A.16) gives

$$x_j = a_{jk}\bar{x}_k \tag{8}$$

Differention gives

$$\frac{\partial x_j}{\partial \bar{x}_i} = a_{jk}\frac{\partial \bar{x}_k}{\partial \bar{x}_i} = a_{jk}\delta_{ik} = a_{ji} \tag{9}$$

(f) From Eq. (4) and (5), we have

$$\frac{\partial \bar{x}_i}{\partial x_j}\frac{\partial x_j}{\partial \bar{x}_k} = a_{ji}a_{jk} = \delta_{ik} \tag{10}$$

The last line follows from Eq. (A.13)