Exercise A.9 Solution

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January 29, 2002

(a) The fluid acceleration is

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} \tag{1}$$

The ith component of which is

$$a_i = \frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \tag{2}$$

(b) The component of acceleration in the direction of the unit vector $\mathbf{n}$ is

$$\mathbf{a} \cdot \mathbf{n} = n_i \frac{\partial u_i}{\partial t} + n_i u_j \frac{\partial u_i}{\partial x_j} \tag{3}$$

(c) The velocity $\mathbf{u}$ can be decomposed as

$$\mathbf{u} = \mathbf{u}^{\parallel} + \mathbf{u}^{\perp} \tag{4}$$

where $\mathbf{u}^{\parallel}$ and $\mathbf{u}^{\perp}$ are parallel and perpendicular to $\mathbf{n}$ respectively. The parallel component is

$$\mathbf{u}^{\parallel} = \mathbf{n}\mathbf{n} \cdot \mathbf{u} \tag{5}$$

and hence (from Eqns. (4) and (5)), the perpendicular component is

$$\begin{aligned}\mathbf{u}^{\perp} =& \mathbf{u} - \mathbf{u}^{\parallel} \\ =& \mathbf{u} - \mathbf{n}\mathbf{n} \cdot \mathbf{u}\end{aligned} \tag{6}$$

the ith component of which is

$$u_i^{\perp} = u_i - n_i n_j u_j = (\delta_{ij} - n_i n_j) u_j \tag{7}$$