Turbulent Flows
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Solution to Exercise 11.25

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Using the pressure decomposition Eq.(11.10) with the Neumann boundary condition Eq.(11.173) at $y = 0$, leads to

$$\frac{1}{\rho}\frac{\partial p^{(r)}}{\partial y} + \frac{1}{\rho}\frac{\partial p^{(s)}}{\partial y} + \frac{1}{\rho}\frac{\partial p^{(h)}}{\partial y} = \nu\frac{\partial^2 v}{\partial y^2}. \tag{1}$$

Here, if zero-normal-gradient (inviscid) boundary conditions are applied for $p^{(r)}$ and $p^{(s)}$ as described bellow Eq.(11.173), the first two terms drop. This leads, together with Eq.(11.174), to a boundary condition for the homogeneous part of the fluctuating pressure,

$$\left.\frac{1}{\rho}\frac{\partial p^{(h)}}{\partial y}\right|_{y=0} = \nu\frac{u_\tau}{\delta_\nu^2}\hat{v}\exp\left(i\kappa_1\frac{x}{\delta_\nu} + i\kappa_3\frac{z}{\delta_\nu}\right). \tag{2}$$

An ansatz for $p^{(h)}$, which satisfies Eq.(2), is

$$p^{(h)} = C^{-1}\rho\nu\frac{u_\tau}{\delta_\nu^2}\hat{v}\exp\left(i\kappa_1\frac{x}{\delta_\nu} + i\kappa_3\frac{z}{\delta_\nu} + Cy\right), \tag{3}$$

where C is a constant to be determined. Inserting Eq.(3) into the Laplace equation $\nabla^2 p^{(h)} = 0$, given after Eq.(11.12), results in the two solutions

$$C_{1,2} = \pm\frac{\sqrt{\kappa_1^2+\kappa_3^2}}{\delta_\nu}. \tag{4}$$

With C_2, the expected exponential decay with increasing y is obtained,

$$p^{(h)} = -\rho\nu\frac{u_\tau}{\delta_\nu}\frac{\hat{v}}{\sqrt{\kappa_1^2+\kappa_3^2}}\exp\left(i\kappa_1\frac{x}{\delta_\nu} + i\kappa_3\frac{z}{\delta_\nu} - \sqrt{\kappa_1^2+\kappa_3^2}\frac{y}{\delta_\nu}\right). \tag{5}$$

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