Turbulent Flows
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Solution to Exercise 11.28

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Since b is a positive constant, Eq.(11.195) can be rewritten,

$$\Delta y = b \min(y, c/b\,\delta). \tag{1}$$

This leads to

$$\Delta y = \begin{cases} by \text{ for } y \leq c/b\,\delta, \\ c\delta \text{ for } c/b\,\delta < y. \end{cases} \tag{2}$$

Inserting Eq.(2) into the approximate equality of Eq.(11.196) and using $b > c$ gives

$$\begin{aligned} N_y &\approx \int_{y_p}^{c/b\,\delta} \frac{dy}{by} + \int_{c/b\,\delta}^{\delta} \frac{dy}{c\delta} \\ &= \frac{1}{b}\left[\ln\left(\frac{c\delta}{b}\right) - \ln(y_p)\right] + \frac{1}{c\delta}\left(\delta - \frac{c\delta}{b}\right) \\ &= \frac{1}{b}\ln\left(\frac{c\delta}{by_p}\right) + \frac{1}{c} - \frac{1}{b} \\ &= \frac{1}{c} + \frac{1}{b}\left[\ln\left(\frac{c\delta}{by_p}\right) - 1\right], \end{aligned} \tag{3}$$

which is equivalent to the second part of Eq.(11.196). With $y_p = d/b\,\delta$, where d is a positive constant with $d < c$, Eq.(3) will reduce to $N_y = N_y(b, c, d) =$ constant. However, with $y_p = 50\delta_\nu$ we obtain

$$N_y \approx \frac{1}{c} + \frac{1}{b}\left[\ln\left(\frac{c\delta}{50b\delta_\nu}\right) - 1\right]. \tag{4}$$

And with the relation $\mathrm{Re}_\tau = \delta/\delta_\nu \approx 0.09\mathrm{Re}^{0.88}$ for the high-Reynolds-number limit (see Section 7.5.1),

$$\begin{aligned} N_y &\approx \frac{1}{c} + \frac{1}{b}\left[\ln\left(\frac{c}{50b}\right) + \ln(0.09) + \ln\left(\mathrm{Re}^{0.88}\right) - 1\right] \\ &\sim \frac{0.88}{b}\ln(\mathrm{Re}) \end{aligned} \tag{5}$$

is resulting.