

Turbulent Flows
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Solution to Exercise 11.28

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Since b is a positive constant, Eq.(11.195) can be rewritten,

$$\Delta y = b \min(y, c/b \delta). \quad (1)$$

This leads to

$$\Delta y = \begin{cases} by & \text{for } y \leq c/b \delta, \\ c\delta & \text{for } c/b \delta < y. \end{cases} \quad (2)$$

Inserting Eq.(2) into the approximate equality of Eq.(11.196) and using $b > c$ gives

$$\begin{aligned} N_y &\approx \int_{y_p}^{c/b\delta} \frac{dy}{by} + \int_{c/b\delta}^{\delta} \frac{dy}{c\delta} \\ &= \frac{1}{b} \left[\ln \left(\frac{c\delta}{b} \right) - \ln(y_p) \right] + \frac{1}{c\delta} \left(\delta - \frac{c\delta}{b} \right) \\ &= \frac{1}{b} \ln \left(\frac{c\delta}{by_p} \right) + \frac{1}{c} - \frac{1}{b} \\ &= \frac{1}{c} + \frac{1}{b} \left[\ln \left(\frac{c\delta}{by_p} \right) - 1 \right], \end{aligned} \quad (3)$$

which is equivalent to the second part of Eq.(11.196). With $y_p = d/b \delta$, where d is a positive constant with $d < c$, Eq.(3) will reduce to $N_y = N_y(b, c, d) =$ constant. However, with $y_p = 50\delta_\nu$ we obtain

$$N_y \approx \frac{1}{c} + \frac{1}{b} \left[\ln \left(\frac{c\delta}{50b\delta_\nu} \right) - 1 \right]. \quad (4)$$

And with the relation $\text{Re}_\tau = \delta/\delta_\nu \approx 0.09\text{Re}^{0.88}$ for the high-Reynolds-number limit (see Section 7.5.1),

$$\begin{aligned} N_y &\approx \frac{1}{c} + \frac{1}{b} \left[\ln \left(\frac{c}{50b} \right) + \ln(0.09) + \ln(\text{Re}^{0.88}) - 1 \right] \\ &\sim \frac{0.88}{b} \ln(\text{Re}) \end{aligned} \quad (5)$$

is resulting.

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