

Turbulent Flows
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Cambridge University Press (2000)
Solution to Exercise 12.24

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Date: 06/05/03

From the Navier-Stokes equations (Eq.(2.35)), we get

$$\frac{D\mathbf{U}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{U}. \quad (1)$$

Following a fluid particle, from Navier-Stokes equations we get

$$\frac{dU_i^*}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\nabla^2 U_i^*. \quad (2)$$

From the GLM equation (Eq.(12.110)), we get

$$dU_i^*(t) = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i} dt + G_{ij}(U_j^*(t) - \langle U_j\rangle) dt + (C_0\varepsilon)^{\frac{1}{2}} dW_i(t), \quad (3)$$

where the coefficient $G_{ij}(\mathbf{x}, t)$ (which is evaluated at $\mathbf{X}^*(t)$) depends on the local values of $\langle u_i u_j\rangle$, ε and $\partial\langle U_i\rangle/\partial x_j$.

From Eq. 3, we get

$$\frac{dU_i^*}{dt} = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i} + G_{ij}(U_j^* - \langle U_j\rangle) + (C_0\varepsilon)^{\frac{1}{2}}\dot{W}_i, \quad (4)$$

where $\dot{\mathbf{W}}$ is white noise (see Eq. (J.29)).

Subtracting Eq. 2 from Eq. 4, we get

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x_i} + \nu\nabla^2 U_i^* = G_{ij}(U_j^* - \langle U_j\rangle) + (C_0\varepsilon)^{\frac{1}{2}}\dot{W}_i. \quad (5)$$

So the Generalized Langevin Model amounts to modelling the fluctuating pressure gradient and viscous terms as

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x_i} + \nu\nabla^2 U_i = \tilde{A}_i(\mathbf{U}, \mathbf{x}, t) \equiv G_{ij}(U_j - \langle U_j\rangle) + (C_0\varepsilon)^{\frac{1}{2}}\dot{W}_i. \quad (6)$$

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