Turbulent Flows
Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise 13.18

Prepared by: Christian Bauer *Date:* 11/12/17

From the definition of the residual stress tensor (13.90) we obtain

$$\begin{aligned}\tau_{ij}^R &= \overline{U_iU_j} - \overline{U}_i\overline{U}_j \\ &= \overline{(\overline{U}_i + u_i')(\overline{U}_j + u_j')} - \overline{U}_i\overline{U}_j \\ &= \underbrace{\overline{\overline{U}_i\overline{U}_j} - \overline{U}_i\overline{U}_j}_{L_{ij}} + \underbrace{\overline{\overline{U}_iu_j'} + \overline{\overline{U}_ju_i'}}_{C_{ij}} + \underbrace{\overline{u_i'u_j'}}_{R_{ij}} \end{aligned} \tag{1}$$

Expression (13.103) on the other hand gives

$$\begin{aligned}\tau_{ij}^\kappa &= \overline{U_iU_j} - \overline{\overline{U}_i\overline{U}_j} \\ &= \overline{(\overline{U}_i + u_i')(\overline{U}_j + u_j')} - \overline{\overline{U}_i\overline{U}_j} \\ &= \underbrace{\overline{\overline{U}_iu_j'} + \overline{\overline{U}_ju_i'}}_{C_{ij}} + \underbrace{\overline{u_i'u_j'}}_{R_{ij}} \end{aligned} \tag{2}$$

The difference between the two terms gives the Leonard stress (13.109)

$$\tau_{ij}^R - \tau_{ij}^\kappa = \overline{\overline{U}_i\overline{U}_j} - \overline{U}_i\overline{U}_j = L_{ij}. \tag{3}$$