

Turbulent Flows
 Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise 13.18

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Date: 11/12/17

From the definition of the residual stress tensor (13.90) we obtain

$$\begin{aligned}
 \tau_{ij}^R &= \overline{U_i U_j} - \overline{U_i} \overline{U_j} \\
 &= \overline{(\overline{U_i} + u'_i)(\overline{U_j} + u'_j)} - \overline{U_i} \overline{U_j} \\
 &= \underbrace{\overline{U_i} \overline{U_j} - \overline{U_i} \overline{U_j}}_{L_{ij}} + \underbrace{\overline{U_i u'_j} + \overline{U_j u'_i}}_{C_{ij}} + \underbrace{\overline{u'_i u'_j}}_{R_{ij}}
 \end{aligned} \tag{1}$$

Expression (13.103) on the other hand gives

$$\begin{aligned}
 \tau_{ij}^\kappa &= \overline{U_i U_j} - \overline{\overline{U_i} \overline{U_j}} \\
 &= \overline{(\overline{U_i} + u'_i)(\overline{U_j} + u'_j)} - \overline{\overline{U_i} \overline{U_j}} \\
 &= \underbrace{\overline{U_i u'_j} + \overline{U_j u'_i}}_{C_{ij}} + \underbrace{\overline{u'_i u'_j}}_{R_{ij}}
 \end{aligned} \tag{2}$$

The difference between the two terms gives the Leonard stress (13.109)

$$\tau_{ij}^R - \tau_{ij}^\kappa = \overline{\overline{U_i} \overline{U_j}} - \overline{U_i} \overline{U_j} = L_{ij}. \tag{3}$$

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