

Turbulent Flows
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Solution to Exercise 3.1

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In general, if $Q(U)$ is a function of U , the mean of $Q(U)$ is given by Eq.(3.20)

$$\langle Q(U) \rangle \equiv \int_{-\infty}^{\infty} Q(V)f(V) dV. \quad (1)$$

Then, we have

$$\langle a \rangle = \int_{-\infty}^{\infty} af(V) dV = a \int_{-\infty}^{\infty} f(V) dV = a, \quad (2)$$

$$\langle aQ \rangle = \int_{-\infty}^{\infty} aQ(V)f(V) dV = a \int_{-\infty}^{\infty} Q(V)f(V) dV = a\langle Q \rangle \quad (3)$$

and

$$\begin{aligned} \langle Q + R \rangle &= \int_{-\infty}^{\infty} (Q(V) + R(V))f(V) dV \\ &= \int_{-\infty}^{\infty} Q(V)f(V) dV + \int_{-\infty}^{\infty} R(V)f(V) dV \\ &= \langle Q \rangle + \langle R \rangle. \end{aligned} \quad (4)$$

Since $\langle Q \rangle$ and $\langle R \rangle$ are constants, by the equation (2), we get $\langle \langle Q \rangle \rangle = \langle Q \rangle$ and $\langle \langle Q \rangle \langle R \rangle \rangle = \langle Q \rangle \langle R \rangle$.

By the equations (3) and (4), the mean of the fluctuation in Q is calculated by

$$\langle q \rangle \equiv \langle Q - \langle Q \rangle \rangle = \langle Q + \langle -Q \rangle \rangle = \langle Q \rangle + \langle -Q \rangle = \langle Q \rangle - \langle Q \rangle = 0. \quad (5)$$

Since $\langle R \rangle$ is a constant, by the equations (3) and (5), $\langle q \langle R \rangle \rangle = \langle q \rangle \langle R \rangle = 0$.

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