

Turbulent Flows
Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise 3.30

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Based on chain rules, we have

$$\frac{du(t+s)}{d(t+s)} = \frac{\partial u(t+s)}{\partial t} = \frac{\partial u(t+s)}{\partial s}. \quad (1)$$

$$\frac{d^2u(t+s)}{d^2(t+s)} = \frac{\partial^2 u(t+s)}{\partial t^2} = \frac{\partial^2 u(t+s)}{\partial s^2}. \quad (2)$$

So we have

$$\begin{aligned} B(s) &= \langle \dot{u}(t)\dot{u}(t+s) \rangle \\ &= \left\langle \frac{du(t)}{dt} \frac{du(t+s)}{d(t+s)} \right\rangle \\ &= \left\langle \frac{\partial}{\partial t} \left(u(t) \frac{\partial u(t+s)}{\partial t} \right) - u(t) \frac{\partial^2 u(t+s)}{\partial t^2} \right\rangle \\ &= \frac{\partial}{\partial t} \langle u(t) \frac{\partial u(t+s)}{\partial t} \rangle - \langle u(t) \frac{\partial^2 u(t+s)}{\partial t^2} \rangle \end{aligned} \quad (3)$$

Since $u(t)$ is a statistically stationary process

$$\frac{\partial}{\partial t} \langle u(t) \frac{\partial u(t+s)}{\partial t} \rangle = 0. \quad (4)$$

So it can be shown that

$$\begin{aligned} B(s) &= -\langle u(t) \frac{\partial^2 u(t+s)}{\partial s^2} \rangle \\ &= -\langle \frac{\partial^2}{\partial s^2} (u(t)u(t+s)) \rangle \\ &= -\frac{\partial^2}{\partial s^2} \langle (u(t)u(t+s)) \rangle \\ &= -\frac{d^2}{ds^2} \langle (u(t)u(t+s)) \rangle \\ &= -\frac{d^2 R(s)}{ds^2} \end{aligned} \tag{5}$$

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