

**Turbulent Flows**  
Stephen B. Pope  
*Cambridge University Press* (2000)

**Solution to Exercise 3.33**

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*Date:* 2/7/03

a) According to Eq.(3.119)

$$\langle \dot{u}(t) | u(t) = v \rangle = \langle \dot{u} \rangle + \frac{\langle \dot{u}u \rangle}{\langle u^2 \rangle} (v - \langle u \rangle) \quad (1)$$

From Exercise 3.27,  $\langle \dot{u}u \rangle = 0$ , so

$$\begin{aligned} \langle \dot{u}(t) | u(t) = v \rangle &= \langle \dot{u} \rangle \\ &= \frac{d \langle u \rangle}{dt} \\ &= 0, \end{aligned} \quad (2)$$

since  $u(t)$  is statistically stationary.

b) Similarly, according to Eq.(3.119)

$$\begin{aligned} \langle \ddot{u}(t) | u(t) = v \rangle &= \langle \ddot{u} \rangle + \frac{\langle \ddot{u}u \rangle}{\langle u^2 \rangle} (v - \langle u \rangle) \\ &= \frac{d \langle \dot{u} \rangle}{dt} - \frac{\langle \dot{u}^2 \rangle}{\langle u^2 \rangle} v \\ &= -v \frac{\langle \dot{u}^2 \rangle}{\langle u^2 \rangle} \end{aligned} \quad (3)$$

The second line follows because  $\langle u \rangle$  is zero and  $\langle \ddot{u}u \rangle = -\langle \dot{u}^2 \rangle$ , see Exercise 3.27.

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