Turbulent Flows
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Solution to Exercise 3.33

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a) According to Eq.(3.119)

$$\langle \dot{u}(t)|u(t)=v\rangle = \langle \dot{u}\rangle + \frac{\langle \dot{u}u\rangle}{\langle u^2\rangle}(v-\langle u\rangle) \tag{1}$$

From Exercise 3.27, $\langle \dot{u}u\rangle = 0$, so

$$\begin{aligned} \langle \dot{u}(t)|u(t)=v\rangle &= \langle \dot{u}\rangle \\ &= \frac{d\langle u\rangle}{dt} \\ &= 0, \end{aligned} \tag{2}$$

since $u(t)$ is statistically stationary.

b) Similarly, according to Eq.(3.119)

$$\begin{aligned} \langle \ddot{u}(t)|u(t)=v\rangle &= \langle \ddot{u}\rangle + \frac{\langle \ddot{u}u\rangle}{\langle u^2\rangle}(v-\langle u\rangle) \\ &= \frac{d\langle \dot{u}\rangle}{dt} - \frac{\langle \dot{u}^2\rangle}{\langle u^2\rangle}v \\ &= -v\frac{\langle \dot{u}^2\rangle}{\langle u^2\rangle} \end{aligned} \tag{3}$$

The second line follows because $\langle u\rangle$ is zero and $\langle \ddot{u}u\rangle = -\langle \dot{u}^2\rangle$, see Exercise 3.27.