

Turbulent Flows
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Cambridge University Press (2000)

Solution to Exercise 3.34

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Date: 5/11/19

With the definition of the two-point correlation,

$$R_{ij}(\mathbf{r}, \mathbf{x}, t) \stackrel{(3.160)}{=} \left\langle u_i(\mathbf{x}, t) u_j(\mathbf{x} + \mathbf{r}, t) \right\rangle \quad (1)$$

$$\begin{aligned} &= \left\langle u_j(\mathbf{x} + \mathbf{r}, t) u_i(\mathbf{x} + \mathbf{r} - \mathbf{r}, t) \right\rangle \\ &\stackrel{\mathbf{x}' := \mathbf{x} + \mathbf{r}}{=} \left\langle u_j(\mathbf{x}', t) u_i(\mathbf{x}' - \mathbf{r}, t) \right\rangle = R_{ji}(-\mathbf{r}, \mathbf{x}', t) \end{aligned} \quad (2)$$

Every statistic (here, in particular, $R_{ij}(\mathbf{r}, \mathbf{x}, t)$) evaluating a statistically homogeneous field does not depend on the point where it is evaluated:

$$\begin{aligned} R_{ij}(\mathbf{r}, \mathbf{x}_1, t) &= R_{ij}(\mathbf{r}, \mathbf{x}_2, t) \quad \forall \mathbf{x}_1, \mathbf{x}_2 \\ \Leftrightarrow R_{ij}(\mathbf{r}, \mathbf{x}, t) &= R_{ij}(\mathbf{r}, t) \end{aligned} \quad (3)$$

Analogous for $R_{ji}(-\mathbf{r}, \mathbf{x}', t)$:

$$\begin{aligned} R_{ji}(-\mathbf{r}, \mathbf{x}'_1, t) &= R_{ji}(-\mathbf{r}, \mathbf{x}'_2, t) \quad \forall \mathbf{x}'_1, \mathbf{x}'_2 \\ \Leftrightarrow R_{ji}(-\mathbf{r}, \mathbf{x}', t) &= R_{ji}(-\mathbf{r}, t) \end{aligned} \quad (4)$$

Since we derived

$$R_{ji}(-\boldsymbol{r}, \boldsymbol{x}', t) = R_{ij}(\boldsymbol{r}, \boldsymbol{x}, t) \quad (2)$$

for an arbitrary point \boldsymbol{x} , we obtain

$$R_{ji}(-\boldsymbol{r}, t) = R_{ij}(\boldsymbol{r}, t) . \quad (5)$$

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