**Turbulent Flows**
Stephen B. Pope
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# Solution to Exercise 3.34

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With the definition of the two-point correlation,

$$\begin{aligned} R_{ij}(\boldsymbol{r},\boldsymbol{x},t) &\overset{(3.160)}{=} \Big\langle u_i\big(\boldsymbol{x},t\big)u_j\big(\boldsymbol{x}+\boldsymbol{r},t\big)\Big\rangle & (1)\\ &= \Big\langle u_j\big(\boldsymbol{x}+\boldsymbol{r},t\big)u_i\big(\boldsymbol{x}+\boldsymbol{r}-\boldsymbol{r},t\big)\Big\rangle \\ &\overset{\boldsymbol{x}':=\boldsymbol{x}+\boldsymbol{r}}{=} \Big\langle u_j\big(\boldsymbol{x}',t\big)u_i\big(\boldsymbol{x}'-\boldsymbol{r},t\big)\Big\rangle = R_{ji}(-\boldsymbol{r},\boldsymbol{x}',t) & (2)\end{aligned}$$

Every statistic (here, in particular, $R_{ij}(\boldsymbol{r},\boldsymbol{x},t)$) evaluating a statistically homogeneous field does not depend on the point where it is evaluated:

$$\begin{aligned} R_{ij}(\boldsymbol{r},\boldsymbol{x}_1,t) &= R_{ij}(\boldsymbol{r},\boldsymbol{x}_2,t)\,\forall\,\boldsymbol{x}_1,\boldsymbol{x}_2 \\ \Leftrightarrow R_{ij}(\boldsymbol{r},\boldsymbol{x},t) &= R_{ij}(\boldsymbol{r},t) & (3)\end{aligned}$$

Analogous for $R_{ji}(-\boldsymbol{r},\boldsymbol{x}',t)$:

$$\begin{aligned} R_{ji}(-\boldsymbol{r},\boldsymbol{x}_1',t) &= R_{ji}(-\boldsymbol{r},\boldsymbol{x}_2',t)\,\forall\,\boldsymbol{x}_1'\boldsymbol{x}_2' \\ \Leftrightarrow R_{ji}(-\boldsymbol{r},\boldsymbol{x}',t) &= R_{ji}(-\boldsymbol{r},t) & (4)\end{aligned}$$

Since we derived

$$R_{ji}(-\boldsymbol{r}, \boldsymbol{x}', t) = R_{ij}(\boldsymbol{r}, \boldsymbol{x}, t) \tag{2}$$

for an arbitrary point $\boldsymbol{x}$, we obtain

$$R_{ji}(-\boldsymbol{r}, t) = R_{ij}(\boldsymbol{r}, t)\,. \tag{5}$$