

**Turbulent Flows**  
Stephen B. Pope  
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**Solution to Exercise 5.13**

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The Cauchy-Schwarz inequality states

$$2ab \leq a^2 + b^2, \{a, b\} \in \mathbb{R}_0^+ \quad (1)$$

To apply the upper inequality, one must ensure that the fluctuating velocities are non-negative. We take the absolute value:

$$\begin{aligned} 2|u||v| &\leq |u|^2 + |v|^2 \\ \stackrel{u, v \in \mathbb{R}}{\Leftrightarrow} |uv| &\leq \left( \frac{u^2}{2} + \frac{v^2}{2} \right) \\ \stackrel{\langle \cdot \rangle}{\Leftrightarrow} \langle |uv| \rangle &\leq \left\langle \frac{1}{2}(u^2 + v^2) \right\rangle \\ \Leftrightarrow |\langle uv \rangle| &\leq \frac{1}{2} \left( \langle u^2 \rangle + \langle v^2 \rangle \right) \leq \frac{1}{2} \left( \langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle \right) = k \quad (2) \end{aligned}$$

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