

Turbulent Flows
 Stephen B. Pope
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Solution to Exercise 5.25

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Starting with the definition of ε (Eq. (5.128)) we have,

$$\varepsilon \equiv 2\nu \langle s_{ij} s_{ij} \rangle. \quad (1)$$

Using the definition of s_{ij} from Eq. (5.130) we have,

$$\begin{aligned} s_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \\ s_{ij} s_{ij} &= \frac{1}{4} \left\{ 2 \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \frac{\partial u_j}{\partial x_i} \right) \right\} \\ s_{ij} s_{ij} &= \frac{1}{2} \left\{ \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \right\}. \end{aligned} \quad (2)$$

Using Eq. (2) in Eq. (1) and using definition of $\tilde{\varepsilon}$ (Eq. (5.159)) and product rule of differentiation we obtain,

$$\begin{aligned} \varepsilon &= \tilde{\varepsilon} + \nu \left\langle \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right\rangle \\ &= \tilde{\varepsilon} + \nu \left\langle \frac{\partial}{\partial x_j} \left(u_i \frac{\partial u_j}{\partial x_i} \right) - u_i \frac{\partial^2 u_j}{\partial x_i \partial x_j} \right\rangle. \end{aligned}$$

Using the fact that $\nabla \cdot \mathbf{u} = 0$ for a constant property flow and the product rule of differentiation in the above equation,

$$\varepsilon = \tilde{\varepsilon} + \nu \left\langle \frac{\partial}{\partial x_j} \left(u_i \frac{\partial u_j}{\partial x_i} \right) \right\rangle$$

$$\begin{aligned}
&= \tilde{\varepsilon} + \nu \left\langle \frac{\partial}{\partial x_j} \left(\frac{\partial(u_i u_j)}{\partial x_i} - u_j \frac{\partial u_i}{\partial x_i} \right) \right\rangle \\
&= \tilde{\varepsilon} + \nu \frac{\partial^2 \langle u_i u_j \rangle}{\partial x_i \partial x_j}.
\end{aligned} \tag{3}$$

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