

Turbulent Flows
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Solution to Exercise 5.4

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From Eq.(5.22) we obtain

$$\begin{aligned}\frac{\partial \langle U \rangle}{\partial x} &= \frac{\partial}{\partial x} (U_0 f(\eta)) \\ &= f \frac{dU_0}{dx} + U_0 f' \left(-\frac{\eta}{x} \right) \\ &= -\frac{U_0}{x} (f + f' \eta) = -\frac{U_0}{x} (f\eta)',\end{aligned}\tag{1}$$

where we have used Eq. (5.13) to substitute for dU_0/dx .

When multiplied by r/U_0 , Eq. (5.21) is

$$\begin{aligned}0 &= \frac{r}{U_0} \frac{\langle U \rangle}{\partial x} + \frac{1}{U_0} \frac{\partial}{\partial r} (rV) \\ &= -\eta (f\eta)' + x \frac{\partial}{\partial r} \left(\frac{r}{x} \frac{V}{U_0} \right) \\ &= -\eta (f\eta)' + \frac{\partial}{\partial \eta} \left(\eta \frac{V}{U_0} \right).\end{aligned}\tag{2}$$

Since the first term is independent of x , $h \equiv V/U_0$ must also be independent of x . Thus we obtain

$$\eta (f\eta)' = (h\eta)'. \tag{3}$$

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