Turbulent Flows
Stephen B. Pope
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Solution to Exercise 5.4

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From Eq.(5.22) we obtain

$$\begin{aligned} \frac{\partial \langle U \rangle}{\partial x} &= \frac{\partial}{\partial x}\left(U_0 f(\eta)\right) \\ &= f\frac{dU_0}{dx} + U_0 f' \left(-\frac{\eta}{x}\right) \\ &= -\frac{U_0}{x}(f + f'\eta) = -\frac{U_0}{x}(f\eta)', \end{aligned} \tag{1}$$

where we have used Eq. (5.13) to substitute for dU_0/dx.

When multiplied by r/U_0, Eq. (5.21) is

$$\begin{aligned} 0 &= \frac{r}{U_0}\frac{\langle U \rangle}{\partial x} + \frac{1}{U_0}\frac{\partial}{\partial r}(rV) \\ &= -\eta(f\eta)' + x\frac{\partial}{\partial r}\left(\frac{r}{x}\frac{V}{U_0}\right) \\ &= -\eta(f\eta)' + \frac{\partial}{\partial \eta}\left(\eta\frac{V}{U_0}\right). \end{aligned} \tag{2}$$

Since the first term is independent of x, $h \equiv V/U_0$ must also be independent of x. Thus we obtain

$$\eta(f\eta)' = (h\eta)'. \tag{3}$$