Turbulent Flows
Stephen B. Pope
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Solution to Exercise 6.34

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Denote the turbulent kinetic energy arising from motions of wavenumbers in the interval (a, b) by

$$k_{(a,b)} = \int_a^b E(\kappa)\mathrm{d}\kappa. \tag{1}$$

Then the fraction of energy arising from motions of wavenumber greater than κ is given by $k_{(\kappa,\infty)}/k$. Let us first calculate this fraction, making use of the Kolmogorov spectrum (Eq.6.239):

$$\begin{aligned} k_{(\kappa,\infty)} &= C\varepsilon^{2/3}\int_\kappa^\infty \tilde{\kappa}^{-2/3}\mathrm{d}\tilde{\kappa} \\ &= \frac{3}{2}C\varepsilon^{2/3}\kappa^{-2/3} \\ &\approx \frac{3}{2}Ck\left(\kappa L\right)^{-2/3}, \end{aligned} \tag{2}$$

where in the last step the definition $L \equiv k^{3/2}/\varepsilon$ of the lengthscale characterizing the large eddies is used.

Note that the total turbulent kinetic energy can (evidently) be decomposed in parts representing wavenumbers smaller and greater than κ:

$$k = k_{(0,\kappa)} + k_{(\kappa,\infty)}. \tag{3}$$

Combining the results of Eqs.(2) and (3), it is derived that

$$\begin{aligned} 1 - \frac{k_{(0,\kappa)}}{k} &= \frac{k_{(\kappa,\infty)}}{k} \\ &\approx \frac{3}{2}\cdot 1.5 \cdot 0.43^{2/3}\cdot\left(\kappa L_{11}\right)^{-2/3} \\ &\approx 1.28\left(\kappa L_{11}\right)^{-2/3}, \end{aligned} \tag{4}$$

where for the numeric result it is used that the Kolmogorov constant $C = 1.5$ and that for large Reynolds number the fraction L_{11}/L tends to 0.43.