

**Turbulent Flows**  
Stephen B. Pope  
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**Solution to Exercise 6.34**

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Denote the turbulent kinetic energy arising from motions of wavenumbers in the interval  $(a, b)$  by

$$k_{(a,b)} = \int_a^b E(\kappa) d\kappa. \quad (1)$$

Then the fraction of energy arising from motions of wavenumber greater than  $\kappa$  is given by  $k_{(\kappa,\infty)}/k$ . Let us first calculate this fraction, making use of the Kolmogorov spectrum (Eq.6.239):

$$\begin{aligned} k_{(\kappa,\infty)} &= C\varepsilon^{2/3} \int_{\kappa}^{\infty} \tilde{\kappa}^{-2/3} d\tilde{\kappa} \\ &= \frac{3}{2} C\varepsilon^{2/3} \kappa^{-2/3} \\ &\approx \frac{3}{2} Ck (\kappa L)^{-2/3}, \end{aligned} \quad (2)$$

where in the last step the definition  $L \equiv k^{3/2}/\varepsilon$  of the lengthscale characterizing the large eddies is used.

Note that the total turbulent kinetic energy can (evidently) be decomposed in parts representing wavenumbers smaller and greater than  $\kappa$ :

$$k = k_{(0,\kappa)} + k_{(\kappa,\infty)}. \quad (3)$$

Combining the results of Eqs.(2) and (3), it is derived that

$$\begin{aligned} 1 - \frac{k_{(0,\kappa)}}{k} &= \frac{k_{(\kappa,\infty)}}{k} \\ &\approx \frac{3}{2} \cdot 1.5 \cdot 0.43^{2/3} \cdot (\kappa L_{11})^{-2/3} \\ &\approx 1.28 (\kappa L_{11})^{-2/3}, \end{aligned} \quad (4)$$

where for the numeric result it is used that the Kolmogorov constant  $C = 1.5$  and that for large Reynolds number the fraction  $L_{11}/L$  tends to 0.43.

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