Turbulent Flows
Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise 6.4

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Eq.(6.46) is

$$\begin{aligned} g(r,t) &= f(r,t) + \tfrac{1}{2} r \frac{\partial}{\partial r} f(r,t) \\ &= f(r,t) + \tfrac{1}{2} \frac{\partial}{\partial r} [r f(r,t)] - \tfrac{1}{2} f(r,t) \\ &= \tfrac{1}{2} \left(f(r,t) + \frac{\partial}{\partial r} [r f(r,t)] \right) \end{aligned} \tag{1}$$

The transverse integral scale is given by Eq.(6.50), using the Eq.(1)

$$\begin{aligned} L_{22}(t) &= \tfrac{1}{2} \int_0^\infty \left(f(r,t) + \frac{\partial}{\partial r} [r f(r,t)] \right) \mathrm{d}r \\ &= \tfrac{1}{2} \int_0^\infty f(r,t)\, \mathrm{d}r + \tfrac{1}{2} \left[r f(r,t) \right]_0^\infty \\ &= \tfrac{1}{2} \int_0^\infty f(r,t)\, \mathrm{d}r \\ &= \tfrac{1}{2} L_{22}(t) \end{aligned} \tag{2}$$

The term $\frac{1}{2} [r f(r,t)]_0^\infty$ becomes zero from the fact that $f(r,t)$ approaches zero faster than $1/r$.

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