

Turbulent Flows
Stephen B. Pope
Cambridge University Press (2000)

Solution to Exercise 6.6

Prepared by: Shi Jin

Date: 3/5/03

a) From Eq.(6.46),

$$g(r, t) = f(r, t) + \frac{1}{2}r \frac{\partial}{\partial r} f(r, t), \quad (1)$$

we have

$$\begin{aligned} g'(r, t) &= f'(r, t) + \frac{1}{2}f'(r, t) + \frac{1}{2}rf''(r, t) \\ &= \frac{3}{2}f'(r, t) + \frac{1}{2}rf''(r, t), \end{aligned} \quad (2)$$

so

$$\begin{aligned} g''(r, t) &= \frac{3}{2}f''(r, t) + \frac{1}{2}f''(r, t) + \frac{1}{2}rf'''(r, t) \\ &= 2f''(r, t) + \frac{1}{2}rf'''(r, t). \end{aligned} \quad (3)$$

b) According to Eq.(6.57)

$$\begin{aligned} \lambda_g(t) &= \left[-\frac{1}{2}g''(0) \right]^{-\frac{1}{2}} \\ &= \left\{ -\frac{1}{2} \left[2f''(0, t) + \frac{1}{2}rf'''(0, t) \right] \right\}^{-\frac{1}{2}} \\ &= \left[-f''(0) \right]^{-\frac{1}{2}} \\ &= \frac{\lambda_f(t)}{\sqrt{2}}. \end{aligned} \quad (4)$$

Notice that $f'''(0, t) = 0$ because $f(r)$ is an even function.

c) From Eqs.(6.56) and (6.68) we have

$$\left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle = \frac{2u'^2}{\lambda_f^2} = \frac{u'^2}{\lambda_g^2}, \quad (5)$$

and hence from Eq.(5.169)

$$\left\langle \left(\frac{\partial u_1}{\partial x_2} \right)^2 \right\rangle = 2 \left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle, \quad (6)$$

we obtain

$$\left\langle \left(\frac{\partial u_1}{\partial x_2} \right)^2 \right\rangle = \frac{2u'^2}{\lambda_g^2}. \quad (7)$$

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/1.0> or send a letter to Creative Commons, 559 Nathan Abbott Way, Stanford, California 94305, USA.