Turbulent Flows
Stephen B. Pope
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Solution to Exercise 7.12

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From Ex. 7.11 we know that $\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}x} = \frac{\mathrm{d}p_w}{\mathrm{d}x} = \text{const}$. Since the pressure drop Δp is defined as a positive quantity, this leads to

$$\frac{\Delta p}{L} = -\frac{\mathrm{d}p_w}{\mathrm{d}x} \overset{(7.101)}{=} 2\frac{\tau_w}{R} = 4\frac{\tau_w}{D},$$

and thus

$$f \overset{(7.97)}{=} \frac{\Delta p D}{\frac{1}{2}\rho \bar{U}^2 L} = \frac{4\tau_w}{\frac{1}{2}\rho \bar{U}^2} \overset{(7.15)}{=} 4C_f.$$

Using the definition $u_\tau \equiv \sqrt{\tau_w/\rho}$, this can be rewritten as

$$f = 8\left(\frac{u_\tau}{\bar{U}}\right)^2,$$

and thus

$$\frac{u_\tau}{\bar{U}} = \sqrt{\frac{f}{8}}.$$