**Turbulent Flows**
Stephen B. Pope
*Cambridge University Press* (2000)

# Solution to Exercise 7.16

*Prepared by:* Michael Stumpf *Date:* 12/17/17

The wall-normal derivative of the mean streamwise velocity profile in the logarithmic region is

$$\begin{aligned}\frac{\mathrm{d}\langle U\rangle}{\mathrm{d}y} &\overset{(7.53)}{=} \frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{u_\tau}{\kappa}\ln\left(\frac{yu_\tau}{\nu}\right)+B\right)\\ &= \frac{u_\tau}{\kappa}\frac{\mathrm{d}}{\mathrm{d}y}\ln\left(\frac{yu_\tau}{\nu}\right)\\ &= \frac{u_\tau}{\kappa}\frac{1}{y},\end{aligned}$$

which, when evaluated at $y=\bar{y}$, gives

$$\left.\frac{\mathrm{d}\langle U\rangle}{\mathrm{d}y}\right|_{y=\bar{y}} = \frac{u_\tau}{\kappa}\frac{1}{\bar{y}} \equiv \frac{\bar{U}}{\delta}.$$

Rearranging leads to

$$\frac{\bar{y}}{\delta} = \frac{1}{\kappa}\frac{u_\tau}{\bar{U}} \overset{(7.104)}{=} \frac{1}{\kappa}\sqrt{\frac{f}{8}} \approx 0.86\sqrt{f}.$$

Scaled in wall units, the equation can be rewritten as

$$\begin{aligned}\frac{\bar{y}}{\delta_v} &= \frac{\delta}{\delta_v}\frac{1}{\kappa}\sqrt{\frac{f}{8}} \overset{(7.27)}{=} \frac{\mathrm{Re}_\tau}{\kappa}\sqrt{\frac{f}{8}}\\ &\overset{(7.26)}{=} \frac{1}{\kappa}\frac{\delta u_\tau}{\nu}\sqrt{\frac{f}{8}}\\ &= \frac{1}{\kappa}\frac{\delta\bar{U}}{\nu}\frac{u_\tau}{\bar{U}}\sqrt{\frac{f}{8}}\\ &\overset{(7.104)}{=} \frac{1}{\kappa}\frac{\delta\bar{U}}{\nu}\frac{f}{8}\\ &\overset{(7.1)}{=} \frac{\mathrm{Re}f}{16\kappa} \approx 0.15\mathrm{Re}f.\end{aligned}$$

Solving Eq. (7.98) iteratively for the two bulk Reynolds numbers given, and subsequent evaluation of Eq. (7.111) and Eq. (7.112) yields

$$\mathrm{Re} = 10^4 : f \approx 0.03089, \qquad \frac{\bar{y}}{\delta} \approx 0.151, \qquad \frac{\bar{y}}{\delta_v} \approx 46.3,$$
$$\mathrm{Re} = 10^6 : f \approx 0.01165, \qquad \frac{\bar{y}}{\delta} \approx 0.093, \qquad \frac{\bar{y}}{\delta_v} \approx 1747.0.$$

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