

Turbulent Flows
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Solution to Exercise 7.17

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The scaled bulk velocity $\frac{\bar{U}}{u_\tau}$ can be obtained for the log law in the fully rough regime from Eq. (7.119):

$$\frac{\bar{U}}{u_\tau} = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R r \left(\frac{1}{\kappa} \ln \left(\frac{y}{s} \right) + B_2 \right) dr d\theta.$$

Substituting $r = R - y$, $dr = -dy$ and subsequent integration by parts yields

$$\begin{aligned} \frac{\bar{U}}{u_\tau} &= \frac{1}{\kappa} \frac{2}{R^2} \int_R^0 (y - R) \ln \left(\frac{y}{s} \right) dy + B_2 \\ &= \frac{1}{\kappa} \frac{2}{R^2} \int_R^0 (y - R) \ln \left(\frac{y}{R} \right) dy, \\ &= -\frac{1}{\kappa} \frac{2}{R^2} \left(\left[\ln \left(\frac{y}{s} \right) \left(\frac{1}{2}y^2 - Ry \right) \right]_{y=R}^{y=0} - \int_R^0 \left(\frac{1}{2}y - R \right) dy \right) + B_2 \\ &= \frac{1}{\kappa} \frac{2}{R^2} \left[\ln \left(\frac{y}{s} \right) \left(\frac{1}{2}y^2 - Ry \right) - \frac{1}{4}y^2 + Ry \right]_{y=R}^{y=0} + B_2 \\ &= \frac{1}{\kappa} \ln \left(\frac{R}{s} \right) + B_2 - \frac{3}{2\kappa}. \end{aligned}$$

From Eq. (7.104), we know that

$$\begin{aligned} f &= 8 \left(\frac{\bar{U}}{u_\tau} \right)^{-2} \\ &= 8 \left[\frac{1}{\kappa} \ln \left(\frac{R}{s} \right) + B_2 - \frac{3}{2\kappa} \right]^{-2}, \end{aligned}$$

which can be reexpressed using $\kappa = 0.41, B_2 = 8.5$ as

$$f = \left(\frac{\sqrt{8}}{\frac{1}{\kappa} \ln(10) \log_{10} \left(\frac{R}{s} \right) + B_2 - \frac{3}{2\kappa}} \right)^2$$

$$\approx \frac{1}{[1.99 \log_{10} \left(\frac{R}{s} \right) + 1.71]^2}.$$

Evaluating the asymptotic friction factors for the values of R/s given in Fig. 7.23 yields very good agreement with the experimental data, as can be seen from the following table.

Table 1: Asymptotic values of the friction factor, computed from Eq. (7.124) for the values of R/s given in Fig. 7.23.

R/s	15	31	60	126	252	507
f	0.061	0.046	0.036	0.0288	0.024	0.020

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