

Turbulent Flows
Stephen B. Pope
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Solution to Exercise 7.25

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From Eq.(7.190), we have

$$\frac{\bar{D}\langle u_i u_j \rangle}{\bar{D}t} + \frac{\partial}{\partial x_k} \langle u_i u_j u_k \rangle = \mathcal{P}_{ij} + \Pi_{ij} + \nu \langle u_i \nabla^2 u_j + u_j \nabla^2 u_i \rangle, \quad (1)$$

from Eq.(7.191), we have

$$\nu \langle u_i \nabla^2 u_j + u_j \nabla^2 u_i \rangle = -\varepsilon_{ij} + \nu \nabla^2 \langle u_i u_j \rangle, \quad (2)$$

from Eq.(7.192), we have

$$\Pi_{ij} = \mathcal{R}_{ij} - \frac{\partial T_{kij}^{(p)}}{\partial x_k}. \quad (3)$$

Substituting Eq. 2 and Eq. 3 into Eq. 1, we get

$$\frac{\bar{D}\langle u_i u_j \rangle}{\bar{D}t} + \frac{\partial}{\partial x_k} \left(\langle u_i u_j u_k \rangle - \nu \frac{\partial \langle u_i u_j \rangle}{\partial x_k} + T_{kij}^{(p)} \right) = \mathcal{P}_{ij} + \mathcal{R}_{ij} - \varepsilon_{ij}, \quad (4)$$

i.e.

$$\frac{\bar{D}\langle u_i u_j \rangle}{\bar{D}t} + \frac{\partial}{\partial x_k} T_{kij} = \mathcal{P}_{ij} + \mathcal{R}_{ij} - \varepsilon_{ij}, \quad (5)$$

where the Reynolds-stress flux T_{kij} is

$$T_{kij} = T_{kij}^{(u)} + T_{kij}^{(p)} + T_{kij}^{(\nu)}, \quad (6)$$

with

$$T_{kij}^{(u)} \equiv \langle u_i u_j u_k \rangle, \quad T_{kij}^{(\nu)} = -\nu \frac{\partial \langle u_i u_j \rangle}{\partial x_k}. \quad (7)$$

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