Turbulent Flows
Stephen B. Pope
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Solution to Exercise 7.25

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From Eq.(7.190), we have

$$\frac{\bar{D}\langle u_i u_j\rangle}{\bar{D}t} + \frac{\partial}{\partial x_k}\langle u_i u_j u_k\rangle = \mathcal{P}_{ij} + \Pi_{ij} + \nu\langle u_i \nabla^2 u_j + u_j \nabla^2 u_i\rangle, \tag{1}$$

from Eq.(7.191), we have

$$\nu\langle u_i \nabla^2 u_j + u_j \nabla^2 u_i\rangle = -\varepsilon_{ij} + \nu\nabla^2\langle u_i u_j\rangle, \tag{2}$$

from Eq.(7.192), we have

$$\Pi_{ij} = \mathcal{R}_{ij} - \frac{\partial T^{(p)}_{kij}}{\partial x_k}. \tag{3}$$

Substituting Eq. 2 and Eq. 3 into Eq. 1, we get

$$\frac{\bar{D}\langle u_i u_j\rangle}{\bar{D}t} + \frac{\partial}{\partial x_k}\left(\langle u_i u_j u_k\rangle - \nu\frac{\partial\langle u_i u_j\rangle}{\partial x_k} + T^{(p)}_{kij}\right) = \mathcal{P}_{ij} + \mathcal{R}_{ij} - \varepsilon_{ij}, \tag{4}$$

i.e.

$$\frac{\bar{D}\langle u_i u_j\rangle}{\bar{D}t} + \frac{\partial}{\partial x_k}T_{kij} = \mathcal{P}_{ij} + \mathcal{R}_{ij} - \varepsilon_{ij}, \tag{5}$$

where the Reynolds-stress flux T_{kij} is

$$T_{kij} = T^{(u)}_{kij} + T^{(p)}_{kij} + T^{(\nu)}_{kij}, \tag{6}$$

with

$$T^{(u)}_{kij} \equiv \langle u_i u_j u_k\rangle, \qquad T^{(\nu)}_{kij} = -\nu\frac{\partial\langle u_i u_j\rangle}{\partial x_k}. \tag{7}$$