

Turbulent Flows
Stephen B. Pope
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Solution to Exercise 7.30

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The definition of ε_{ij} is

$$\varepsilon_{ij} \equiv 2\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right\rangle. \quad (1)$$

For very small y (very close to the wall), the fluctuating velocity components can be written as Taylor series of the forms (see section 7.1.6)

$$u = b_1 y + c_1 y^2 + \dots, \quad (2)$$

$$v = c_2 y^2 + \dots, \quad (3)$$

$$w = b_3 y + c_3 y^2. \quad (4)$$

Now, at the wall ($y=0$), all components of \mathbf{u} are zero, as are the derivatives $\partial u_i/\partial x_1$ and $\partial u_i/\partial x_3$. The only non-zero derivatives are $\partial u_i/\partial x_2 = \partial u_i/\partial y$. Thus we obtain

$$\begin{aligned} \varepsilon_{ij} &= 2\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right\rangle = 2\nu \left\langle \frac{\partial u_i}{\partial y} \frac{\partial u_j}{\partial y} \right\rangle \\ &= \nu \frac{\partial^2 \langle u_i u_j \rangle}{\partial y^2} - \langle u_i \frac{\partial^2 u_j}{\partial y^2} \rangle - \langle u_j \frac{\partial^2 u_i}{\partial y^2} \rangle \\ &= \nu \frac{\partial^2 \langle u_i u_j \rangle}{\partial y^2}. \end{aligned} \quad (5)$$

$$\varepsilon_{11} = 2\nu \left\langle \frac{\partial u}{\partial y} \frac{\partial u}{\partial y} \right\rangle = 2\nu \langle b_1^2 \rangle,$$

$$\varepsilon_{22} = 2\nu \left\langle \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} \right\rangle = 0,$$

$$\begin{aligned}
\varepsilon_{33} &= 2\nu \left\langle \frac{\partial w}{\partial y} \frac{\partial w}{\partial y} \right\rangle = 2\nu \langle b_3^2 \rangle, \\
\varepsilon_{12} = \varepsilon_{21} &= 2\nu \left\langle \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \right\rangle = 0, \\
\varepsilon_{13} = \varepsilon_{31} &= 2\nu \left\langle \frac{\partial u}{\partial y} \frac{\partial w}{\partial y} \right\rangle = 2\nu \langle b_1 b_3 \rangle, \\
\varepsilon_{23} = \varepsilon_{32} &= 2\nu \left\langle \frac{\partial v}{\partial y} \frac{\partial w}{\partial y} \right\rangle = 0.
\end{aligned} \tag{6}$$

In short, ε_{11} , ε_{33} , ε_{13} and ε_{31} are non-zero at the wall, and other components are zero at the wall.

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