

Figure 7.1: Sketch of (a) channel flow (b) pipe flow and (c) flat-plate boundary layer.

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Figure 7.2: Mean velocity profiles in fully-developed turbulent channel flow from the DNS of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750

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Figure 7.3: Profiles of the viscous shear stress, and the Reynolds shear stress in turbulent channel flow: DNS data of Kim *et al.* (1987): dashed line, Re = 5,600; solid line, Re = 13,750.



Figure 7.4: Profiles of the fractional contributions of the viscous and Reynolds stresses to the total stress. DNS data of Kim *et al.* (1987): dashed lines, Re = 5,600; solid lines, Re = 13,750.



Figure 7.5: Near-wall profiles of mean velocity from the DNS data of Kim *et al.*: dashed line, Re = 5,600; solid line, Re = 13,750; dot-dashed line, $u^+ = y^+$.

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Figure 7.6: Near-wall profiles of mean velocity: solid line, DNS data of Kim *et al.*: Re = 13,750; dot-dashed line, $u^+ = y^+$; dashed line, the log law, Eqs. (7.43)–(7.44).



Figure 7.7: Mean velocity profiles in fully-developed turbulent channel flow measured by Wei and Willmarth (1989): \circ , Re₀ = 2, 970; \Box , Re₀ = 14, 914; Δ , Re₀ = 22, 776; ∇ , Re₀ = 39, 582; line, the log law, Eqs. (7.43)–(7.44).



Figure 7.9: Mean velocity defect in turbulent channel flow. Solid line, DNS of Kim *et al.* (1987), Re = 13,750; dashed line, log law, Eqs. (7.43)–(7.44).



Figure 7.10: Skin friction coefficient $c_f \equiv \tau_w/(\frac{1}{2}\rho U_0^2)$ against Reynolds number (Re = $2\bar{U}\delta/\nu$) for channel flow: symbols, experimental data compiled by Dean (1978); solid line, from Eq. (7.55); dashed line, laminar friction law $c_f = 16/(3Re)$.



Figure 7.11: Outer-to-inner lengthscale ratio $\delta/\delta_{\nu} = \text{Re}_{\tau}$ for turbulent channel flow as a function of Reynolds number (obtained from Eq. 7.55).



Figure 7.12: Outer-to-inner velocity scale ratios for turbulent channel flow as functions of Reynolds number (obtained from Eq. 7.55): solid line, \bar{U}/u_{τ} ; dashed line U_0/u_{τ} .



Figure 7.13: Regions and layers in turbulent channel flow as functions of Reynolds number.

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Figure 7.14: Reynolds stresses and kinetic energy normalized by friction velocity against y^+ from DNS of channel flow at Re = 13, 750 (Kim *et al.* 1987).



Figure 7.15: Profiles of Reynolds stresses normalized by turbulent kinetic energy from DNS of channel flow at Re = 13,750 (Kim *et al.* 1987).



Figure 7.16: Profiles of the ratio of production to dissipation $(\mathcal{P}/\varepsilon)$, normalized mean shear rate $(\mathcal{S}k/\varepsilon)$, and shear stress correlation coefficient (ρ_{uv}) from DNS of channel flow at Re = 13,750 (Kim *et al.* 1987).

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Figure 7.17: Profiles of Reynolds stresses and kinetic energy normalized by friction velocity in the viscous wall region of turbulent channel flow: DNS data of Kim *et al.* (1987) Re = 13, 750.



Figure 7.18: Turbulent kinetic energy budget in the viscous wall region of channel flow: terms in Eq. (7.64) normalized by viscous scales. From the DNS data of Kim *et al.* (1987) Re = 13, 750.

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Figure 7.19: Profiles of r.m.s. velocity measured in channel flow at different Reynolds numbers by Wei and Willmarth (1989). Open symbols: $u'/u_{\tau} = \langle u^2 \rangle^{\frac{1}{2}}/u_{\tau}$; \bigcirc , Re₀ = 2,970; \square , Re₀ = 14,914; \triangle , Re₀ = 22,776; \bigtriangledown , Re₀ = 39,582. Solid symbols: $v'/u_{\tau} = \langle v^2 \rangle^{\frac{1}{2}}/u_{\tau}$ at the same Reynolds numbers.



Figure 7.20: Mean velocity profiles in fully-developed turbulent pipe flow. Symbols, experimental data of Zagarola and Smits (1997) at six Reynolds numbers (Re $\approx 32 \times 10^3$, 99×10^3 , 409×10^3 , 1.79×10^6 , 7.71×10^6 , 29.9×10^6). Solid line, log law with $\kappa = 0.436$ and B = 6.13; dashed line, log law with $\kappa = 0.41$, B = 5.2.



Figure 7.21: Mean velocity profiles in fully-developed turbulent pipe flow. Symbols, experimental data of Zagarola and Smits (1997) for y/R < 0.1, for the same values of Re as in Fig. 7.20. Line, log law with $\kappa = 0.436$ and B = 6.13.



Figure 7.25: Normalized velocity and shear stress profiles from the Blasius solution for the zero-pressure-gradient laminar boundary layer on a flat plate: y is normalized by $\delta_x \equiv x/\text{Re}_x^{\frac{1}{2}} = (x\nu/U_0)^{\frac{1}{2}}$.



Figure 7.26: Profiles of mean velocity, shear stress and intermittency factor in a zero-pressure gradient turbulent boundary layer, $\text{Re}_{\theta} = 8,000$. From the experimental data of Klebanoff (1954).



Figure 7.27: Mean velocity profiles in wall units. Circles, boundarylayer experiments of Klebanoff (1954), $\text{Re}_{\theta} = 8,000$; dashed line, boundary-layer DNS of Spalart (1988), $\text{Re}_{\theta} = 1,410$; dot-dashed line, channel flow DNS of Kim *et al.* (1987), Re = 13,750; solid line, van Driest's law of the wall, Eqs. (7.144)–(7.145).



Figure 7.28: Mean velocity profile in a turbulent boundary layer showing the law of the wake. Symbols, experimental data of Klebanoff (1954); dashed line, log law ($\kappa = 0.41, B = 5.2$); dot-dashed line, wake contribution $\Pi w(y/\delta)/\kappa$ ($\Pi = 0.5$); solid line, sum of log law and wake contribution (Eq. 7.148).



Figure 7.29: Velocity defect law. Symbols, experimental data of Klebanoff (1954); dashed line, log law; solid line, sum of log law and wake contribution $\Pi w(y/\delta)/\kappa$.

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Figure 7.30: Turbulent viscosity and mixing length deduced from direct numerical simulations of a turbulent boundary layer (Spalart 1988). Solid line, ν_T from DNS; dot-dash line, ℓ_m from DNS; dashed line ℓ_m and ν_T according to van Driest's specification (Eq. 7.145).



Figure 7.31: Log-log plot of mean velocity profiles in turbulent pipe flow at six Reynolds number (from left to right: Re $\approx 32 \times 10^3$, 99×10^3 , 409×10^3 , 1.79×10^6 , 7.71×10^6 , 29.9×10^6). The scale for u^+ pertains to the lowest Reynolds number: subsequent profiles are shifted down successively by a factor of 1.1. The range shown is the overlap region, $50\delta_{\nu} < y < 0.1 R$. Symbols, experimental data of Zagarola and Smits (1997); dashed lines, log law with $\kappa = 0.436$ and B = 6.13; solid lines, power law (Eq. 7.157) with the power α determined by the best fit to the data



Figure 7.32: Exponent $\alpha = 1/n$ (Eq. 7.158) in the power-law $u^+ = C(y^+)^{\alpha} = C(y^+)^{1/n}$ for pipe flow as a function of Reynolds number.

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Figure 7.33: Profiles of Reynolds stresses and kinetic energy normalized by the friction velocity in a turbulent boundary layer at $\text{Re}_{\theta} =$ 1,410: (a) across the boundary layer (b) in the viscous near-wall region. From the DNS data of Spalart (1988).

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Figure 7.34: Turbulent kinetic energy budget in a turbulent boundary layer at $\text{Re}_{\theta} = 1,410$: terms in Eq. (7.177) (a) normalized as a function of y so that the sum of the squares of the terms is unity (b) normalized by the viscous scales. From the DNS data of Spalart (1988).

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Figure 7.35: Budget of $\langle u^2 \rangle$ in a turbulent boundary layer: conditions and normalization are the same as in Fig. 7.34.

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Figure 7.36: Budget of $\langle v^2 \rangle$ in a turbulent boundary layer: conditions and normalization are the same as in Fig. 7.34.

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Figure 7.37: Budget of $\langle w^2 \rangle$ in a turbulent boundary layer: conditions and normalization are the same as in Fig. 7.34.

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Figure 7.38: Budget of $-\langle uv \rangle$ in a turbulent boundary layer: conditions and normalization are the same as in Fig. 7.34.



Figure 7.39: Normalized dissipation components in a turbulent boundary layer at $\text{Re}_{\theta} = 1,410$: from the DNS data of Spalart (1988) for which $\delta = 650\delta_{\nu}$.



Figure 7.40: Dye streak in a turbulent boundary layer showing the ejection of low-speed near-wall fluid. (From the experiment of Kline *et al.* 1967.)

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Figure 7.42: Sketch of counter-rotating rolls in the near-wall region. (From Holmes $et \ al. 1996$.)



Figure 7.43: Sketch of counter-rotating rolls in the near-wall region. (From Holmes *et al.* 1996.)

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Figure 7.44: The large-scale features of a turbulent boundary layer at $\text{Re}_{\theta} \approx 4,000$. The irregular line—approximating the viscous superlayer—is the boundary between smoke-filled turbulent fluid and clear free-stream fluid. (From the experiment of Falco 1977.)